TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Description Logics

Exercise Sheet 6.1

Dr. Felix Distel
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Exercise 25

Let $\mathcal{I}$ be a fuzzy interpretation and let $\mathcal{I}^{\text{crisp}} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}^{\text{crisp}}})$ be the interpretation defined as

$$\begin{aligned} A^{\mathcal{I}^{\text{crisp}}}(x) &= \mathbb{1}\left(A^{\mathcal{I}}(x)\right) \\ r^{\mathcal{I}^{\text{crisp}}}(x, y) &= \mathbb{1}\left(r^{\mathcal{I}}(x, y)\right) \end{aligned}$$

for all concept names $A \in \mathcal{N}_C$ and all role names $r \in \mathcal{N}_R$. Let D, E be $\otimes$-$\mathcal{EL}^{\neg\sqcup}$ concepts satisfying

$$D^{\mathcal{I}^{\text{crisp}}}(x) = \mathbb{1}\left(D^{\mathcal{I}}(x)\right), \qquad E^{\mathcal{I}^{\text{crisp}}}(x) = \mathbb{1}\left(E^{\mathcal{I}}(x)\right).$$

Prove that

a) $(D \sqcup E)^{\mathcal{I}^{\text{crisp}}}(x) = \mathbb{1}\left((D \sqcup E)^{\mathcal{I}}(x)\right)$

b) $(D \to E)^{\mathcal{I}^{\text{crisp}}}(x) = \mathbb{1}\left((D \to E)^{\mathcal{I}}(x)\right)$

c) $(\neg D)^{\mathcal{I}^{\text{crisp}}}(x) = \mathbb{1}\left((\neg D)^{\mathcal{I}}(x)\right)$

holds for all $x \in \Delta^{\mathcal{I}}$.

Exercise 26

Let $\mathcal{A}$ be an $\otimes$-$\mathcal{EL}^{\neg\sqcup}$ ABox and let $\mathcal{I}$ be a model of $\mathcal{A}$. Show that $\mathcal{I}^{\text{crisp}}$ defined as in Exercise 25 is also a model of $\mathcal{A}$.

Exercise 27

Let $\mathcal{A}$ be a $\otimes$-$\mathcal{ALC}$ ABox and let $\mathcal{A}^{\text{crisp}}$ be the ABox obtained from $\mathcal{A}$ by replacing all positive truth degrees in the axioms by 1. Let $\mathcal{I}$ be a *crisp* interpretation. Prove the following:

a) If $\mathcal{I}$ is a model of $\mathcal{A}^{\text{crisp}}$ then $\mathcal{I}$ is a model of $\mathcal{A}$.

b) If $\mathcal{I}$ is a model of $\mathcal{A}$ then $\mathcal{I}$ is a model of $\mathcal{A}^{\text{crisp}}$.