Fuzzy Description Logics

Exercise Sheet 10
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Exercise 40
Check for which of the three fundamental continuous t-norms the following equalities hold. Provide a proof or a counterexample when appropriate.

a) \( \ominus \ominus x = x \)
b) \( x \Rightarrow y = \ominus x \oplus y \)
c) \( x \oplus y = \ominus(\ominus x \otimes \ominus y) \)
d) \( x \otimes \ominus x = 1 \)
e) \( x \otimes (x \Rightarrow y) = x \otimes y \)

Exercise 41
Consider the logic \( \otimes - ALC \) with the product t-norm and \( = \)-axioms. Let \( w \) be a word over the alphabet \( \{1, \ldots, s\} \). Let \( V, U \) be concept names. Define

- a function \( e: \{1, \ldots, s\}^* \rightarrow [0, 1] \), and
- an ontology \( O \),

such that every model \( I \) of \( O \) satisfies

\[
V^I(x) = e(v) \quad \Rightarrow \quad U^I(x) = e(vw)
\]

for all \( x \in \Delta^I \).