



Term Rewriting Systems

Exercise Sheet 2

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Exercise 5

Which of the closure operators defined in the lecture commute? Prove or refute the validity of each of the following equations:

a) $(\overset{\pm}{\rightarrow})^= = (\overline{\rightarrow})^+$

b) $(\rightarrow \cup \overleftarrow{\rightarrow})^+ = \overset{\pm}{\rightarrow} \cup (\overset{\pm}{\rightarrow})^{-1}$

c) $(\overleftarrow{\rightarrow})^+ = (\overset{\pm}{\rightarrow})^{-1}$

Exercise 6

Consider the following reduction relations:

- Let M be a set and 2^M the power set of M . We define the reduction relation \rightarrow_M on 2^M as follows: $A \rightarrow_M B$ iff $B \subsetneq A$.
- We define the reduction relation $\rightarrow_{p,q}$ on the non-negative integers as follows:
 $n \rightarrow_{p,q} n - p$ iff $n > p$, and
 $n \rightarrow_{p,q} n - q$ iff $n > q$.

For each of the following properties, describe those sets M and those non-negative integers p, q such that \rightarrow_M and $\rightarrow_{p,q}$ satisfy the following property:

- terminating,
- Church-Rosser,
- normalising, and
- confluent.

Exercise 7

In the lecture, we defined the set $A := \mathbb{N} \setminus \{0, 1\}$ and the following 'divisibility' relation on A :

$$\rightarrow := \{(m, n) \mid m > n \text{ and there is some } \ell \in A \text{ with } n \cdot \ell = m\}.$$

Prove that $\overset{*}{\leftrightarrow} = A \times A$.

