Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 5

Dr. rer. nat. Rafael Peñaloza / Marcel Lippmann
Summer Semester 2014

Exercise 21

Let ρ and τ be substitutions.

a) Describe the variable range $\mathsf{VRan}(\rho\tau)$.

b) Under what conditions do we have $\rho\tau = \tau$?

c) Under what conditions do we have $\tau\tau = \tau$?

Exercise 22

Let E be a set of identities with some $\ell \approx r \in E$ such that ℓ is a variable or $\mathsf{Var}(r) \not\subseteq \mathsf{Var}(\ell)$. Prove that $\to_E$ is not terminating.

Exercise 23

Let $G_1 := \{f^3(a) \approx a, f^5(a) \approx a\}$ and $G_2 := \{f^4(a) \approx a, f^6(a) \approx a\}$. Consider the congruence closures of G_1 and G_2 and verify whether $f^2(a) \approx a$ holds in G_1 or G_2.

Exercise 24

Let G be a set of ground identities and $\mathsf{CC}(G)$ the congruence closure of G. Complete the proof of Lemma 4.8 of the lecture by proving that $\to_G \subseteq \mathsf{CC}(G)$.