TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 10

Dr. rer. nat. Rafael Peñaloza / Marcel Lippmann
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Exercise 45

Prove that the termination of the TRS $R = \{f(f(x)) \to g(x), g(g(x)) \to f(x)\}$ cannot be proved using a lexicographic path order.

Exercise 46

Prove termination of the following TRS R using a lexicographic path order:

$$R = \{s(x) + (y + z) \to x + (s(s(y)) * z),$$
$$s(x_1) + (x_2 + (x_3 + x_4)) \to x_1 + (x_2 + (x_3 + x_4))\}$$

Exercise 47

Let Σ be finite signature with at least one constant symbol, $>$ a strict partial order on Σ, and $>_{\mathsf{lpo}}$ the lexicographic path order induced by $>$. Prove the following claim: If $>$ is a total order on Σ, then $>_{\mathsf{lpo}}$ is total on ground terms.

Exercise 48

Prove the following claim: If $>$ is a reduction order on $\mathcal{T}(\Sigma, V)$ that is total on ground terms, then $>$ satisfies the subterm property on ground terms, i.e. for each ground term t and position $p \in \mathsf{Pos}(t) \setminus \{\varepsilon\}$, we have $t > t|_p$.

Exercise 49

Complete the proof of Thm. 6.1 of the lecture: Let E be a set of identities over Σ. Prove the following equivalence for all terms $s, t \in T(\Sigma, V)$:

$$s \approx_E t \text{ iff } \mathsf{const}(s) \approx_E \mathsf{const}(t),$$

where $\mathsf{const}(\cdot)$ is a function that replaces every occurrence of a variable x with a constant $a_x \notin \Sigma$.

Exercise 50

Find terms r_1, r_2 such that $\{f(g(x)) \to r_1, g(h(x)) \to r_2\}$ is confluent.

Exercise 51

Compute all critical pairs for the TRS consisting of the following rules:

$$0 + y \to y, \quad s(x) + y \to s(x + y)$$
$$x + 0 \to x, \quad x + s(y) \to s(x + y)$$

Is the system locally confluent? Is it convergent?

Exercise 52

Finish Example 6.8 from the lecture: Show that the TRS $\{f(f(x)) \to g(x), f(g(x)) \to g(f(x))\}$ is terminating and confluent.

Exercise 53

Consider the decision procedure in Corollary 6.7 for the confluence of finite, terminating TRS. Can you establish an upper bound for the runtime of the procedure as a function of the size of the input TRS?

Exercise 54

Consider the system $\{f(x) \to g(x, y)\}$. Does it have any critical pairs? Is the induced rewrite relation confluent? What is going wrong here?