Term Rewriting Systems

Exercise Sheet 10

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Exercise 45
Prove that the termination of the TRS $R = \{ f(f(x)) \rightarrow g(x), g(g(x)) \rightarrow f(x) \}$ cannot be proved using a lexicographic path order.

Exercise 46
Prove termination of the following TRS $R$ using a lexicographic path order:

$$R = \{ s(x) + (y + z) \rightarrow x + (s(y)) \cdot z),$$
$$s(x_1) + (x_2 + (x_3 + x_4)) \rightarrow x_1 + (x_2 + (x_3 + x_4)) \}$$

Exercise 47
Let $\Sigma$ be finite signature with at least one constant symbol, $>$ a strict partial order on $\Sigma$, and $>_{lpo}$ the lexicographic path order induced by $>$. Prove the following claim: If $>$ is a total order on $\Sigma$, then $>_{lpo}$ is total on ground terms.

Exercise 48
Prove the following claim: If $>$ is a reduction order on $T(\Sigma, V)$ that is total on ground terms, then $>$ satisfies the subterm property on ground terms, i.e. for each ground term $t$ and position $p \in Pos(t) \{ \varepsilon \}$, we have $t > t|_p$.

Exercise 49
Complete the proof of Thm. 6.1 of the lecture: Let $E$ be a set of identities over $\Sigma$. Prove the following equivalence for all terms $s, t \in T(\Sigma, V)$:

$$s \approx_E t \text{ iff const}(s) \approx_E \text{ const}(t),$$

where const$(\cdot)$ is a function that replaces every occurrence of a variable $x$ with a constant $a_x \notin \Sigma$.

Exercise 50
Find terms $r_1, r_2$ such that $\{ f(g(x)) \rightarrow r_1, g(h(x)) \rightarrow r_2 \}$ is confluent.
Exercise 51

Compute all critical pairs for the TRS consisting of the following rules:

\[0 + y \rightarrow y, \ s(x) + y \rightarrow s(x + y)\]
\[x + 0 \rightarrow x, \ x + s(y) \rightarrow s(x + y)\]

Is the system locally confluent? Is it convergent?

Exercise 52

Finish Example 6.8 from the lecture: Show that the TRS \{f(f(x)) \rightarrow g(x), f(g(x)) \rightarrow g(f(x))\} is terminating and confluent.

Exercise 53

Consider the decision procedure in Corollary 6.7 for the confluence of finite, terminating TRS. Can you establish an upper bound for the runtime of the procedure as a function of the size of the input TRS?

Exercise 54

Consider the system \{f(x) \rightarrow g(x, y)\}. Does it have any critical pairs? Is the induced rewrite relation confluent? What is going wrong here?