



Term Rewriting Systems

Exercise Sheet 11

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Exercise 55

Consider the following sets of identities:

$$E_1 := \{f(g(f(x))) \approx x\}$$

$$E_2 := \{f(g(f(x))) \approx f(g(x))\}$$

- Apply the basic completion procedure to E_1 and E_2 .
- What happens if the improved completion procedure that also simplifies rules is applied to E_1 ?

Exercise 56

Show that the encompassment quasi-order \sqsupseteq is in fact a quasi-order and that the associated strict order \sqsubset is a well-founded strict order.

Exercise 57

Let \equiv denote the equivalence relation associated to \sqsupseteq , i.e. $s \equiv t$ iff $s \sqsupseteq t$ and $t \sqsupseteq s$. Show that:

- $s \equiv t$ iff s and t are equal up to variable renaming.
- For a given term s , there exist up to variable renaming only finitely many terms t_i such that $s \sqsupseteq t_i$.

Exercise 58

Consider the following completion procedure for ground term rewriting systems:

Input: G_0 , a finite set of ground identities over Σ , $>$, a reduction order that is total on the set of ground terms over Σ .

Procedure: Apply the rules L-SIMPLIFY-RULE, DELETE, and ORIENT, until no more rule is applicable.

Output: A ground term rewriting system.

Show that this procedure

- a) always terminates,
- b) is fair,
- c) is correct, and
- d) never fails.