TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Exercise Sheet 1

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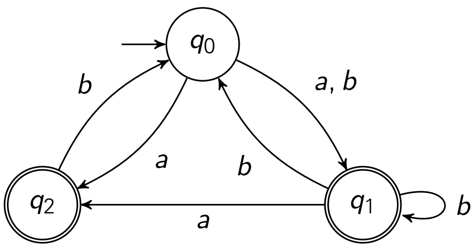
Exercise 1

Let $\Sigma = \{a, b\}$ be an alphabet and $\alpha := a^+b^* + b^+a^*$ a regular expression over Σ. Give a regular expression β for the the complement language of α, i.e. β describes the set of words over Σ that are not expressed by α.

Exercise 2

Let $\mathcal{A}$ be a non-deterministic automaton $\mathcal{A} := (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_1, q_2\})$ with Δ given by the following transition system:



Apply the power-set construction to $\mathcal{A}$ in order to obtain a *deterministic* automaton that accepts the same language as $\mathcal{A}$.

Exercise 3

For a language $L \subseteq \Sigma^*$, the *Nerode right congruence* ρ_L is defined as follows. For $u, v \in \Sigma^*$, we have:

$$u \rho_L v \quad \text{iff} \quad \text{for all } w \in \Sigma^*,\ uw \in L \Leftrightarrow vw \in L.$$

Let $\mathcal{A}_L := (Q_L, \Sigma, q_L, \delta_L, F_L)$ be a deterministic automaton where:

- $Q_L := \{[u]_{\rho_L} \mid u \in \Sigma^*\}$ where $[u]_{\rho_L} := \{v \in \Sigma^* \mid u\rho_L v\}$,
- $q_L := [\varepsilon]_{\rho_L}$ where ε denotes the empty word,
- $\delta_L([u]_{\rho_L}, a) := [ua]_{\rho_L}$ for $u \in \Sigma^*$, $a \in \Sigma$,
- $F_L := \{[u]_{\rho_L} \mid u \in L\}$.

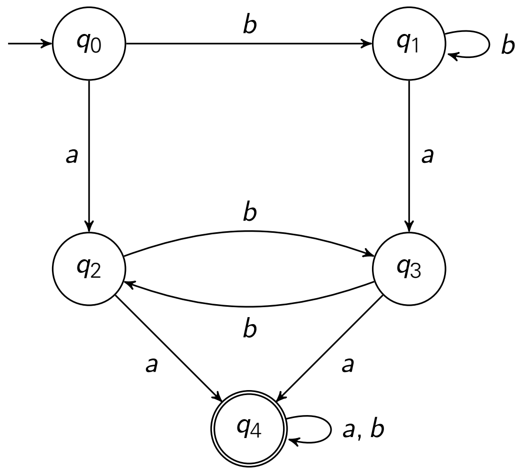
Show the following for regular languages L:

a) $\mathcal{A}_L$ is well-defined.

b) $\mathcal{A}_L$ is minimal (w.r.t. the number of states), i.e. for every deterministic automaton $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ with $L(\mathcal{A}) = L$, we have $|Q_L| \leq |Q|$.

Exercise 4

Let $\mathcal{A}$ be the automaton that accepts words over the alphabet $\Sigma := \{a, b\}$ described by the following transition system:



Construct an automaton $\mathcal{A}'$ such that $L(\mathcal{A}') = L(\mathcal{A})$ and $\mathcal{A}'$ is minimal.

Exercise 5

Prove the following by giving a decision procedure:

a) The *emptiness problem* for regular languages is decidable.

b) The *inclusion problem* for regular languages is decidable.