Automata and Logic

Exercise Sheet 6
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Exercise 35
Let $\Sigma := \{a\}$. Recall from the lecture that $L_{k,n}$ denotes the set of all first-order formulae (over the non-logical symbols $=$, $<$, and $Q_a$) containing $k$ free variables and having quantifier depth at most $n$. For the following combinations of $k, n$, determine a finite set $\Gamma_{k,n}$ such that for every formula $\phi \in L_{k,n}$, there is a formula $\psi \in \Gamma_{k,n}$ with $\phi \equiv \psi$. Determine also the equivalence classes of $\equiv_{k,n}$.

a) $k = 1, n = 0$;
b) $k = 2, n = 0$;
c) $k = 0, n = 1$; and
d) $k = 1, n = 1$.

Exercise 36
Give the formulae $\phi_W$ for each equivalence class $W$ of $\equiv_{2,0}$. Then determine a finite disjunction of formulae $\phi_W$ for $\equiv_{2,0}$-classes which is equivalent to the formulae:

a) true;
b) $\neg(x < y) \lor x = y$; and
c) false.

Exercise 37
Consider the Ehrenfeucht-Fraïssé games on the words

a) $ab$ and $ba$; and
b) $aaabaaa$ and $aabaaa$.

Determine the $k \in \{1, \ldots, 4\}$ such that Player I has a winning strategy in $k$ moves.
Exercise 38

Consider the Ehrenfeucht-Fraïssé games on the words $a_i$ and $a_j$ with $i < j$.

a) Describe an optimal winning strategy for Player I, i.e., a strategy such that Player I wins with a minimal number of moves.

b) Prove that Player I has a winning strategy on $a_i$ and $a_j$ (with $i < j$) in $m$ moves if $i < 2^m - 1$. 