



## Automata and Logic

### Exercise Sheet 6

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#### Exercise 35

Let  $\Sigma := \{a\}$ . Recall from the lecture that  $L_{k,n}$  denotes the set of all first-order formulae (over the non-logical symbols  $=, <, \text{ and } Q_a$ ) containing  $k$  free variables and having quantifier depth at most  $n$ . For the following combinations of  $k, n$ , determine a *finite* set  $\Gamma_{k,n}$  such that for every formula  $\phi \in L_{k,n}$ , there is a formula  $\psi \in \Gamma_{k,n}$  with  $\phi \equiv \psi$ . Determine also the equivalence classes of  $\equiv_{k,n}$ .

- a)  $k = 1, n = 0$ ;
- b)  $k = 2, n = 0$ ;
- c)  $k = 0, n = 1$ ; and
- d)  $k = 1, n = 1$ .

#### Exercise 36

Give the formulae  $\phi_W$  for each equivalence class  $W$  of  $\equiv_{2,0}$ . Then determine a finite disjunction of formulae  $\phi_W$  for  $\equiv_{2,0}$ -classes which is equivalent to the formulae:

- a) true;
- b)  $\neg(x < y) \vee x = y$ ; and
- c) false.

#### Exercise 37

Consider the Ehrenfeucht-Fraïssé games on the words

- a)  $ab$  and  $ba$ ; and
- b)  $aaabaaa$  and  $aabaaa$ .

Determine the  $k \in \{1, \dots, 4\}$  such that Player I has a winning strategy in  $k$  moves.

**Exercise 38**

Consider the Ehrenfeucht-Fraïssé games on the words  $a^i$  and  $a^j$  with  $i < j$ .

- a) Describe an optimal winning strategy for Player I, i.e., a strategy such that Player I wins with a minimal number of moves.
- b) Prove that Player I has a winning strategy on  $a^i$  and  $a^j$  (with  $i < j$ ) in  $m$  moves if  $i < 2^m - 1$ .