



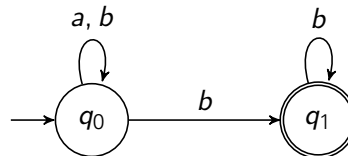
## Automata and Logic

### Exercise Sheet 9

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#### Exercise 48

Let  $\Sigma := \{a, b\}$ , and  $L \subseteq \Sigma^\omega$  be the  $\omega$ -language recognised by the following Büchi automaton:



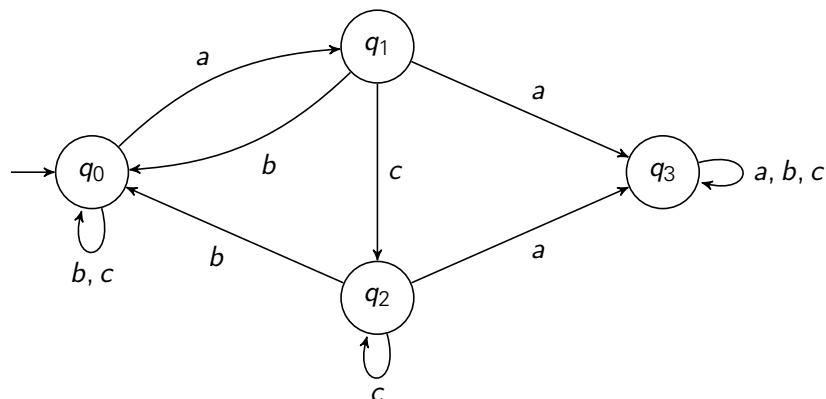
Use the method presented in the lecture to construct a Büchi automaton that recognises the language  $\Sigma^\omega \setminus L$ .

#### Exercise 49

Prove that for every  $\omega$ -regular language  $L$ , there is a Büchi automaton  $\mathcal{A}$  with  $L_\omega(\mathcal{A}) = L$  such that from every state  $q$  of  $\mathcal{A}$ , there are *at most two* transitions using the same alphabet symbol.

#### Exercise 50

Let  $\Sigma := \{a, b, c\}$ . Consider the following transition system:



We derive four Muller automata  $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ , and  $\mathcal{A}_4$  by selecting the sets of final states  $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3$ , and  $\mathcal{F}_4$  as follows:

- a)  $\mathcal{F}_1 := \{\{q_0, q_3\}, \{q_3\}\}$ ;
- b)  $\mathcal{F}_2 := \{\{q_0, q_1\}, \{q_2\}\}$ ;

c)  $\mathcal{F}_3 := \{\{q_0, q_1, q_2\}\}$ ; and

d)  $\mathcal{F}_4 := \{\{q_0\}, \{q_0, q_1\}, \{q_2\}, \{q_0, q_1, q_2\}\}$ .

Determine the  $\omega$ -languages  $L_\omega(\mathcal{A}_1)$ ,  $L_\omega(\mathcal{A}_2)$ ,  $L_\omega(\mathcal{A}_3)$ , and  $L_\omega(\mathcal{A}_4)$ .