TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Exercise Sheet 11

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Exercise 57

Let Σ be an alphabet with arity function, $x, y \in \Sigma_0$, and $U, V, W \subseteq \mathbf{T}_\Sigma$. Prove or refute:

- $U \cdot^x (V \cup W) = (U \cdot^x V) \cup (U \cdot^x W)$;
- $(U \cdot^x V) \cdot^y W = U \cdot^x (V \cdot^y W)$;
- $(U^{*,x})^{*,y} = (U^{*,y})^{*,x}$.

Exercise 58

Example 6.10 from the lecture shows that deterministic RL-tree automata recognise a smaller class of languages than non-deterministic ones. We call an RL-tree automaton $\mathcal{A} := (Q, \Sigma, I, \Delta, F)$ *quasi-deterministic* if

- Δ is a deterministic transition assignment, and
- $I \subseteq Q$ is a *set* of initial states.

Prove or refute:

a) If $L \subseteq \mathbf{T}_\Sigma$ is a *finite* tree language, then there exists a quasi-deterministic tree automaton recognising L.

b) If $L \subseteq \mathbf{T}_\Sigma$ is a *recognisable* tree language, then there exists a quasi-deterministic tree automaton recognising L.

Exercise 59

Devise a polynomial time algorithm that decides the emptiness problem for LR-tree automata.

Exercise 60

Let $\mathcal{A} := (Q, \Sigma, I, \Delta, F)$ be an RL-tree automaton given by:

- $Q := \{1, \ldots, 4\}$;
- $\Sigma := \{g, n, a, b\}$ with $\nu(g) := 2$, $\nu(n) := 1$, and $\nu(a) := \nu(b) := 0$;
- $I := \{1\}$;
- $\Delta_g(1) := \{(1,1), (1,2), (3,4), (4,1)\}$, $\Delta_g(2) := \emptyset$, $\Delta_g(3) := Q \times Q$, $\Delta_g(4) := \{(1,2), (1,4), (2,4), (2,2)\}$;

- $\Delta_n(1) := \{1\}$, $\Delta_n(2) := \{3\}$, $\Delta_n(3) := \{1, 2\}$, $\Delta_n(4) := \{1, 3\}$; and
- $F(a) := \{2\}$, $F(b) := \{2, 3\}$.

Decide whether $L(\mathcal{A}) = \emptyset$ or not.

Exercise 61

Let $\Sigma := \{a, b\}$ be an alphabet with two binary symbols and

$$L := \{t \in \mathbf{T}^{\omega}_{\Sigma} \mid \text{there is a path in } t \text{ containing the symbol } a \text{ only finitely often}\}.$$

a) Is L Rabin-recognisable?

b) Is L Büchi-recognisable?

Exercise 62

Let Σ be an alphabet (with arity function) containing at least two binary symbols f and g. Prove or refute that the ω-tree language $L := \{f(t, t) \mid t \in \mathbf{T}^{\omega}_{\Sigma}\}$ is Büchi-recognisable.