Exercise 57
Let $\Sigma$ be an alphabet with arity function, $x, y \in \Sigma_0$, and $U, V, W \subseteq T_\Sigma$. Prove or refute:

- $U \cdot x (V \cup W) = (U \cdot x V) \cup (U \cdot x W)$;
- $(U \cdot x V)^y W = U \cdot x (V \cdot y W)$;
- $(U^*x)^*y = (U^*y)^*x$.

Exercise 58
Example 6.10 from the lecture shows that deterministic RL-tree automata recognise a smaller class of languages than non-deterministic ones. We call an RL-tree automaton $A := (Q, \Sigma, I, \Delta, F)$ quasi-deterministic if

- $\Delta$ is a deterministic transition assignment, and
- $I \subseteq Q$ is a set of initial states.

Prove or refute:

a) If $L \subseteq T_\Sigma$ is a finite tree language, then there exists a quasi-deterministic tree automaton recognising $L$.

b) If $L \subseteq T_\Sigma$ is a recognisable tree language, then there exists a quasi-deterministic tree automaton recognising $L$.

Exercise 59
Devise a polynomial time algorithm that decides the emptiness problem for LR-tree automata.

Exercise 60
Let $A := (Q, \Sigma, I, \Delta, F)$ be an RL-tree automaton given by:

- $Q := \{1, \ldots, 4\}$;
- $\Sigma := \{g, n, a, b\}$ with $\nu(g) := 2$, $\nu(n) := 1$, and $\nu(a) := \nu(b) := 0$;
- $I := \{1\}$;
- $\Delta_g(1) := \{(1, 1), (1, 2), (3, 4), (4, 1)\}$, $\Delta_g(2) := \emptyset$, $\Delta_g(3) := Q \times Q$, $\Delta_g(4) := \{(1, 2), (1, 4), (2, 4), (2, 2)\}$;
• $\Delta_n(1) := \{1\}$, $\Delta_n(2) := \{3\}$, $\Delta_n(3) := \{1, 2\}$, $\Delta_n(4) := \{1, 3\}$; and
• $F(a) := \{2\}$, $F(b) := \{2, 3\}$.

Decide whether $L(A) = \emptyset$ or not.

**Exercise 61**

Let $\Sigma := \{a, b\}$ be an alphabet with two binary symbols and

$L := \{t \in T^\omega_\Sigma \mid$ there is a path in $t$ containing the symbol $a$ only finitely often$\}$. 

a) Is $L$ Rabin-recognisable?

b) Is $L$ Büchi-recognisable?

**Exercise 62**

Let $\Sigma$ be an alphabet (with arity function) containing at least two binary symbols $f$ and $g$. Prove or refute that the $\omega$-tree language $L := \{f(t, t) \mid t \in T^\omega_\Sigma\}$ is Büchi-recognisable.