Exercise 63
Let $\Sigma$ be an alphabet of binary symbols.

- For each $\omega$-tree $t \in T_\Sigma$, we define a language of $\omega$-words $\text{path}(t) \subseteq \Sigma^\omega$ as follows:
  \[
  \text{path}(t) := \{ \alpha \in \Sigma^\omega \mid \text{there is a path } i_0 i_1 i_2 \ldots \text{ in } t \text{ such that } t(i_0 i_1 \ldots i_n) = \alpha(n) \text{ for all } n \in \mathbb{N} \}.
  \]
  For $B \subseteq T_\Sigma$, let $\text{path}(B) := \bigcup_{t \in B} \text{path}(t)$.

- For a language $L \subseteq \Sigma^\omega$ of $\omega$-words, let $\text{tree}(L) := \{ t \in T_\Sigma \mid \text{path}(t) \subseteq L \}$.

Prove or refute:

- a) $\text{tree}(\text{path}(B)) = B$
- b) $\text{path}(\text{tree}(L)) = L$
- c) If $B$ is Büchi-recognisable, then $\text{path}(B)$ is also Büchi-recognisable.
- d) If $\text{path}(B)$ is Büchi-recognisable, then $B$ is also Büchi-recognisable.
- e) If $L$ is Büchi-recognisable, then $\text{tree}(L)$ is also Büchi-recognisable.
- f) If $\text{tree}(L)$ is Büchi-recognisable, then $L$ is also Büchi-recognisable.

Exercise 64
Let $\Sigma := \{ a, b \}$ and $A$ be the Rabin-automaton $A = (\{ q_0, q_1, q_2 \}, \Sigma, \{ q_2 \}, \Delta, \{ \{ q_0, q_2 \}, \{ q_1 \} \})$ where:

\[
\Delta_a : q_0 \mapsto \{ (q_1, q_1) \} \quad \Delta_b : q_0 \mapsto \{ (q_0, q_0) \} \\
q_1 \mapsto \{ (q_1, q_1) \} \quad q_1 \mapsto \{ (q_0, q_0) \} \\
q_2 \mapsto \{ (q_0, q_1) \} \quad q_2 \mapsto \{ (q_0, q_1) \}
\]

Use the method from the proof of Prop. 7.12 from the lecture to decide whether $L_\omega(A) = \emptyset$ or not.
**Exercise 65**

Let $\Sigma := \{0, 1\}$ and

$$L := \{ t \in T^\omega_\Sigma \mid \text{for every path } p, \text{ if } p \text{ contains the symbol } 0, \text{ then } p \text{ contains the symbol } 1 \text{ only finitely often} \}.$$

Give an S2S-formula $\phi$ such that $L_\omega(\phi) = L$.

**Exercise 66**

For the automaton $A$ from exercise 64, give an S2S-formula $\phi$ with $L_\omega(A) = L_\omega(\phi)$. 