TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Exercise Sheet 12

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Exercise 63

Let Σ be an alphabet of binary symbols.

- For each ω-tree $t \in \mathbf{T}^\omega_\Sigma$, we define a language of ω-words $\mathsf{path}(t) \subseteq \Sigma^\omega$ as follows:

  $\mathsf{path}(t) := \{\alpha \in \Sigma^\omega \mid \text{there is a path } i_0 i_1 i_2 \ldots \text{ in } t \text{ such that } t(i_0 i_1 \ldots i_n) = \alpha(n) \text{ for all } n \in \mathbb{N}\}.$

  For $B \subseteq \mathbf{T}^\omega_\Sigma$, let $\mathsf{path}(B) := \bigcup_{t \in B} \mathsf{path}(t)$.
- For a language $L \subseteq \Sigma^\omega$ of ω-words, let $\mathsf{tree}(L) := \{t \in \mathbf{T}^\omega_\Sigma \mid \mathsf{path}(t) \subseteq L\}$.

Prove or refute:

a) $\mathsf{tree}(\mathsf{path}(B)) = B$

b) $\mathsf{path}(\mathsf{tree}(L)) = L$

c) If B is Büchi-recognisable, then $\mathsf{path}(B)$ is also Büchi-recognisable.

d) If $\mathsf{path}(B)$ is Büchi-recognisable, then B is also Büchi-recognisable.

e) If L is Büchi-recognisable, then $\mathsf{tree}(L)$ is also Büchi-recognisable.

f) If $\mathsf{tree}(L)$ is Büchi-recognisable, then L is also Büchi-recognisable.

Exercise 64

Let $\Sigma := \{a, b\}$ and $\mathcal{A}$ be the Rabin-automaton $\mathcal{A} = (\{q_0, q_1, q_2\}, \Sigma, \{q_2\}, \Delta, \{(\{q_0, q_2\}, \{q_1\})\})$ where:

$$\begin{aligned} \Delta_a\colon\ q_0 &\mapsto \{(q_1, q_1)\} & \Delta_b\colon\ q_0 &\mapsto \{(q_0, q_0)\} \\ q_1 &\mapsto \{(q_1, q_1)\} & q_1 &\mapsto \{(q_0, q_0)\} \\ q_2 &\mapsto \{(q_0, q_1)\} & q_2 &\mapsto \{(q_0, q_1)\} \end{aligned}$$

Use the method from the proof of Prop. 7.12 from the lecture to decide whether $L_\omega(\mathcal{A}) = \emptyset$ or not.

Exercise 65

Let $\Sigma := \{0, 1\}$ and

$$L := \{t \in \mathbf{T}_\Sigma^\omega \mid \text{for every path } p \text{, if } p \text{ contains the symbol 0,}$$
$$\text{then } p \text{ contains the symbol 1 only finitely often}\}.$$

Give an S2S-formula ϕ such that $L_\omega(\phi) = L$.

Exercise 66

For the automaton $\mathcal{A}$ from exercise 64, give an S2S-formula ϕ with $L_\omega(\mathcal{A}) = L_\omega(\phi)$.