TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Automatic Structures

Summer Semester 2016

Exercise Sheet 1 – Word Automatic Structures

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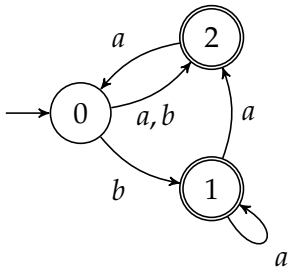
Exercise 1.1 Let $\Sigma := \{a, b\}$, and consider the language

$$L := \{\, w \in \Sigma^* \mid \text{the number of occurences of } a \text{ in } w \text{ is even} \,\}.$$

Prove or refute: L is a regular language.

Exercise 1.2 Consider the non-deterministic finite automaton $M := (\{0,1,2\}, \{a,b\}, \{0\}, \Delta, \{1,2\})$ whose transition relation Δ is given by the following graphical representation.



Construct a finite automaton $\overline{M}$ that accepts the complement of $L(M)$.

Hint. Use the powerset construction to obtain a deterministic finite automaton that accepts the same language as M.

Exercise 1.3 Let Σ be an alphabet. Show that the collection of regular languages over Σ is closed under

(a) union,

(b) intersection,

(c) complement,

(d) concatenation, and

(e) KLEENE-star.

Exercise 1.4 (a) Prove that the collection of automatic relations is closed under universal projection w.r.t. regular languages.

(b) Show that the collection of automatic relations is not closed under concatenation, where the concatenation $R \circ S$ of two k-ary relations R and S over Σ^* is defined as

$$R \circ S := \{\, (u_1 v_1, \ldots, u_k v_k) \mid (u_1, \ldots, u_k) \in R \text{ and } (v_1, \ldots, v_k) \in S \,\}.$$