



Term Rewriting Systems

Exercise Sheet 3

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Exercise 10

Let $\mathbf{ack} : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ be the Ackermann function, i.e.

$$\mathbf{ack}(n, m) = \begin{cases} m + 1 & \text{if } n = 0, \\ \mathbf{ack}(n - 1, 1) & \text{if } m = 0, \\ \mathbf{ack}(n - 1, \mathbf{ack}(n, m - 1)) & \text{otherwise.} \end{cases}$$

Choose an appropriate order on pairs $(n, m) \in \mathbb{N} \times \mathbb{N}$ and use well-founded induction to prove that the function \mathbf{ack} is well-defined, i.e. for each pair of non-negative integers it determines a unique value.

Exercise 11

Prove Lemma 2.21 from the lecture:

If $>_A$ is a strict order on A , and $>_B$ is a strict order on B , then the lexicographic product $>_{A \times B}$ is a strict order on $A \times B$.

Exercise 12

Prove that, for a strict order $>$, its induced multiset order $>_{\text{mul}}$ is a strict order.

Exercise 13

Let $>$ be a strict, decidable order on a finite set A .

- Show that the induced multiset order $>_{\text{mul}}$ is decidable.
- Design a simpler decision procedure for the case when $>$ is a total strict order on A .

Exercise 14

Let $(A, >)$ be a strict order. Prove or refute the following claims.

- If A has a smallest element, then $>$ is well-founded.
- If every non-empty subset of A has a smallest element, then $>$ is well-founded.
- If $>$ is well-founded, then, for each element $a \in A$, there are only finitely many b with $a > b$.
- If $>$ is well-founded, then, for each element $a \in A$, there is an $n_a \in \mathbb{N}$ such that each $>$ -path starting at a is of length at most n_a .
- If $>$ is well-founded, then each non-empty subset of A has a smallest element.