Term Rewriting Systems

Exercise Sheet 4

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Summer Semester 2016

Exercise 15

The reduction relation $\to$ enjoys the *diamond property* if

$$y_1 \leftarrow x \to y_2 \implies \exists z. y_1 \to z \leftarrow y_2.$$

Prove that, if $\to$ enjoys the diamond property, then every element x is either in normal form or does not have a normal form.

Exercise 16

Let $(A, \to_1 \cup \to_2)$ be the reduction system obtained from the reduction systems $(A, \to_1)$ and $(A, \to_2)$ by building the union of the two reduction relations.

Prove or refute: If $\to_1$ and $\to_2$ are confluent, then so is $\to_1 \cup \to_2$.

Exercise 17

Does *strong confluence* imply the following property?

$$y_1 \leftarrow x \to y_2 \implies \exists z. y_1 \stackrel{=}{\to} z \stackrel{=}{\leftarrow} y_2$$

Give a proof or counterexample.

Exercise 18

Consider the terms $s = f(x, g(h(k(k(y)), x), z), h(x, y))$ and $t = g(z, h(x, k(k(y))))$.

Describe $t|_1$, $t|_{1111}$, $t|_{11111}$, $t[s]_2$, and $t[s|_2]_{21}$.

Exercise 19

Prove the second part of Lemma 3.4 by induction on the length of words denoting positions:

If $p \in \text{Pos}(s)$ and $q \in \text{Pos}(t)$, then

$$(s[t]_p)|_{pq} = t|_q$$
$$(s[t]_p)[r]_{pq} = s[t[r]_q]_p$$

Exercise 20

Prove Lemma 3.10:

> $\rightarrow_E$ is closed under substitutions and compatible with Σ-contexts.

Exercise 21

Prove Proposition 3.16:

> Let $\mathcal{A}$ be a Σ-algebra and $\varphi\colon X \to A$ a mapping. Then there exists a unique homomorphism $\phi\colon \mathcal{T}(\Sigma, X) \to \mathcal{A}$ such that $\varphi(x) = \phi(x)$ for all $x \in X$.