



Term Rewriting Systems

Exercise Sheet 5

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Exercise 22

Let ρ and τ be substitutions.

- Describe the variable range $\text{VRan}(\rho\tau)$.
- Under what conditions do we have $\rho\tau = \tau$?
- Under what conditions do we have $\tau\tau = \tau$?

Exercise 23

Let E be a set of identities with some $\ell \approx r \in E$ such that ℓ is a variable or $\text{Var}(r) \not\subseteq \text{Var}(\ell)$.
Prove that \rightarrow_E is not terminating.

Exercise 24

Let $G_1 := \{f^3(a) \approx a, f^5(a) \approx a\}$ and $G_2 := \{f^4(a) \approx a, f^6(a) \approx a\}$. Consider the congruence closures of G_1 and G_2 and verify whether $f^2(a) \approx a$ holds in G_1 or G_2 .

Exercise 25

Let G be a set of ground identities and $\text{CC}(G)$ the congruence closure of G . Complete the proof of Lemma 4.8 of the lecture by proving that $\rightarrow_G \subseteq \text{CC}(G)$.

Exercise 26

Let E be a set of identities. Prove or refute the following claims:

- $\overset{*}{\leftrightarrow}_E = \overset{*}{\rightarrow}_{E \cup E^{-1}}$
- $\overset{*}{\rightarrow}_E = \rightarrow_{E^*}$