



Term Rewriting Systems

Exercise Sheet 7

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Summer Semester 2016

Exercise 30

Why do we need $\vec{\rightarrow}$ in the proof of Theorem 5.7? More precisely, for a Turing machine \mathcal{M} , let

$$\Sigma'_{\mathcal{M}} := \{s_0, \dots, s_n\} \cup \{q_0, \dots, q_p\} \cup \{\vec{\ell}, \overleftarrow{r}\}$$

and let $R'_{\mathcal{M}}$ be a rewrite system obtained from $R_{\mathcal{M}}$ by replacing both \vec{s}_i and \overleftarrow{s}_i with s_i . Give an example of a terminating Turing machine \mathcal{M} for which $R'_{\mathcal{M}}$ does not terminate.

Exercise 31

Prove that the following is *not* a decision procedure for the termination of a ground term rewriting system $R = \{\ell_i \rightarrow r_i \mid 1 \leq i \leq n\}$:

Generate all reduction sequences starting with r_1 .

If one of these sequences yields a term that has r_1 as a subterm, then answer 'non-terminating'. Otherwise, continue with r_2 , etc.

Exercise 32

Prove the 'left version' of Theorem 5.9:

Termination of a finite, left-ground term rewriting system is decidable.

Exercise 33

We define the order $>$ on terms as follows:

$$s > t \text{ if } |s| > |t| \text{ and } |s|_x \geq |t|_x \text{ for all } x \in V.$$

Prove that $>$ is a reduction order on $\mathcal{T}(\Sigma, V)$.