TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 9

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Exercise 41

For each of the following pairs (s, t) of terms, check whether $s \trianglerighteq_{\text{emb}} t$.

a) $f(x) \trianglerighteq_{\text{emb}} a$

b) $f(b) \trianglerighteq_{\text{emb}} a$

c) $g(g(x, y), g(a, f(z))) \trianglerighteq_{\text{emb}} g(y, g(a, z))$

Exercise 42

Prove the following claims:

a) The reduction relation $\xrightarrow{*}_{R_{\text{emb}}}$ given by the TRS

$$R_{\text{emb}} := \{f(x_1, \dots, x_n) \to x_i \mid n \geq 1, f \in \Sigma^{(n)} \text{ and } 1 \leq i \leq n\}$$

and the homeomorphic embedding $\trianglerighteq_{\text{emb}}$ are identical, i.e. $s \xrightarrow{*}_{R_{\text{emb}}} t$ iff $s \trianglerighteq_{\text{emb}} t$.

b) $\trianglerighteq_{\text{emb}}$ is a partial order.

c) $\trianglerighteq_{\text{emb}}$ is well-founded. (Prove this without using Kruskal's Theorem.)

Exercise 43

In the proof of Theorem 5.32, we have used that $\trianglerighteq_{\text{emb}}$ is a well-partial-order. Explain why $\trianglerighteq_{\text{emb}}$ being a well-founded partial order would not have been sufficient.

Exercise 44

In lecture, it was shown that the termination of the TRS $R := \{f(f(x)) \to f(g(f(x)))\}$ cannot be proved using a simplification order.

a) Prove termination using the interpretation method.

b) Is there a polynomial order that can be used to prove termination of R?

Exercise 45

Prove that polynomial orders are simplification orders if the following properties are satisfied.

- The underlying signature Σ contains only function symbols of arity at least 2.
- The domain A does not contain 1, i.e. $A \subseteq \mathbb{N} \setminus \{0, 1\}$.

Are those conditions necessary?

Exercise 46

Prove the first part of Theorem 5.38 of the lecture:

> Let Σ be a finite signature, $s, t \in \mathcal{T}(\Sigma, V)$, and $>_{\mathrm{lpo}}$ be a lexicographic path order.
> We can decide whether $s >_{\mathrm{lpo}} t$ in time polynomial in $|s|$ and $|t|$.

Hint:

First, show that the condition

$$s >_{\mathrm{lpo}} t_j \text{ for all } j \text{ with } 1 \leq j \leq n$$

in (LPO2c) can be replaced with

$$s >_{\mathrm{lpo}} t_j \text{ for all } j \text{ with } i \leq j \leq n \text{ for } i \text{ such that } s_1 = t_1 \ldots s_{i-1} = t_{i-1}, \text{ and } s_i >_{\mathrm{lpo}} t_i.$$

Use this modified condition to prove that the question whether $s >_{\mathrm{lpo}} t$ holds can be decided in time $O(|s| \cdot |t|)$.