TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 10/11

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Summer Semester 2016

Exercise 47

Prove that the termination of the TRS $R = \{f(f(x)) \to g(x),\ g(g(x)) \to f(x)\}$ cannot be proved using a lexicographic path order.

Exercise 48

Prove termination of the following TRS R using a lexicographic path order:

$$\begin{aligned} R = \{&s(x) + (y + z) \to x + (s(s(y)) * z),\\ &s(x_1) + (x_2 + (x_3 + x_4)) \to x_1 + (x_2 + (x_3 + x_4))\} \end{aligned}$$

Exercise 49

Let Σ be finite signature with at least one constant symbol, $>$ a strict partial order on Σ, and $>_{\mathsf{lpo}}$ the lexicographic path order induced by $>$. Prove the following claim:

> If $>$ is a total order on Σ, then $>_{\mathsf{lpo}}$ is total on ground terms.

Exercise 50

Prove the following claim:

> If $>$ is a reduction order on $\mathcal{T}(\Sigma, V)$ that is total on ground terms, then $>$ satisfies the subterm property on ground terms, i.e. for each ground term t and position $p \in \mathsf{Pos}(t) \setminus \{\varepsilon\}$, we have $t > t|_p$.

Exercise 51

Complete the proof of Theorem 6.1 of the lecture:

> Let E be a set of identities over Σ. Prove the following equivalence for all terms $s, t \in T(\Sigma, V)$:
>
> $$s \approx_E t \text{ iff } \mathsf{const}(s) \approx_E \mathsf{const}(t),$$
>
> where const$(\cdot)$ is a function that replaces every occurrence of a variable x with a constant $a_x \notin \Sigma$.

Exercise 52

Find terms r_1, r_2 such that $\{f(g(x)) \to r_1,\ g(h(x)) \to r_2\}$ is confluent.

Exercise 53

Compute all critical pairs for the TRS consisting of the following rules:

$$0 + y \to y,\ \ s(x) + y \to s(x + y)$$
$$x + 0 \to x,\ \ x + s(y) \to s(x + y)$$

Is the system locally confluent? Is it convergent?

Exercise 54

Finish Example 6.8 of the lecture:

> Show that the TRS $\{f(f(x)) \to g(x),\ f(g(x)) \to g(f(x))\}$ is terminating and confluent.

Exercise 55

Consider the decision procedure in Corollary 6.7 for the confluence of finite, terminating TRS. Can you establish an upper bound for the runtime of the procedure as a function on the size of the input TRS?

Exercise 56

Consider the system $\{f(x) \to g(x, y)\}$. Does it have any critical pairs? Is the induced rewrite relation confluent? What is going wrong here?

Exercise 57

Show that the TRS

$$\{(x * y) * (y * z) \to y,\ x * ((x * y) * z) \to x * y,\ (x * (y * z)) * z \to y * z\}$$

is confluent.

Exercise 58

Consider a generalisation of the TRS of Example 7.1:

$$E := \{x + s^n(0) \approx s^n(x) \mid n \geq 0\} \cup \{s(x + y) \approx x + s(y)\}.$$

Find a reduction order $>$ with

$$x + s^n(0) > s^n(x) \qquad \text{for } n \geq 0$$
$$s(x + y) > x + s(y)$$

to prove termination.