



## Term Rewriting Systems

### Exercise Sheet 12

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#### Exercise 59

Consider the following sets of identities:

$$E_1 := \{f(g(f(x))) \approx x\}$$

$$E_2 := \{f(g(f(x))) \approx f(g(x))\}$$

- Apply the basic completion procedure to  $E_1$  and  $E_2$ .
- What happens if the improved completion procedure that also simplifies rules is applied to  $E_1$ ?

#### Exercise 60

Show that the encompassment quasi-order  $\sqsupseteq$  is in fact a quasi-order and that the associated strict order  $\sqsupset$  is a well-founded strict order.

#### Exercise 61

Let  $\equiv$  denote the equivalence relation associated to  $\sqsupseteq$ , i.e.  $s \equiv t$  iff  $s \sqsupseteq t$  and  $t \sqsupseteq s$ . Show that:

- $s \equiv t$  iff  $s$  and  $t$  are equal up to variable renaming.
- For a given term  $s$ , there exist up to variable renaming only finitely many terms  $t_i$  such that  $s \sqsupseteq t_i$ .

#### Exercise 62

Consider the following completion procedure for ground term rewriting systems:

**Input:**  $G_0$ , a finite set of ground identities over  $\Sigma$ ,  $>$ , a reduction order that is total on the set of ground terms over  $\Sigma$ .

**Procedure:** Apply the rules L-SIMPLIFY-RULE, DELETE, and ORIENT, until no more rule is applicable.

**Output:** A ground term rewriting system.

Show that this procedure

- always terminates,
- is fair,
- is correct, and
- never fails.