Term Rewriting Systems
Exercise Sheet 13
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Summer Semester 2016

Exercise 63
a) Consider the following set of identities:
\[ E := \{ f(f(x, y), z) \approx f(x, f(y, z)), \; f(x, x) \approx x, \; f(f(x, y), x) \approx x \} \]
Apply the rules of the improved completion procedure to \( E \). Use a strategy that resembles the basic completion procedure, but simplifies rules as follows: upon adding new rules, simplify old ones by means of L-SIMPLIFY-RULE and R-SIMPLIFY-RULE.
Consider the proof
\[ P := (f(x, f(y, f(y, x))), \; f(x, f(f(y, y), x)), \; f(x, f(y, x)), \; f(f(x, y), x), \; x). \]
Construct a rewrite proof \( P' \) in \( \mathcal{R}_\omega \) with \( P \triangleright E P' \) using the proof of Lemma 7.21.

b) Consider the following set of identities:
\[ E := \{ x + (y + z) \approx (x + y) + z, \; f(x) + f(y) \approx f(x + y) \} \]
Apply the completion procedure described above to input \( E \) and the polynomial order induced by
\[ P_f(X) = X + 1, \; P_+(X, Y) = XY^2. \]

Exercise 64
The semi-decision procedure described in the proof of Theorem 7.22 of the lecture is rather inefficient: For the input \( s \approx_E t \), all \( R_i \)-normal forms of \( s \) and \( t \) are computed in the \( i \)th iteration of the repeat-loop. Show that the following modification of the procedure still yields a semi-decision procedure for the word problem:
- Begin with \( s_0 := s \) and \( t_0 := t \).
- After the \( i \)th repeat-loop, compute one arbitrary \( R_i \)-normal form \( s_i \) of \( s_{i-1} \) and one arbitrary \( R_i \)-normal form \( t_i \) of \( t_{i-1} \).
- Output ‘yes’ \( (s \approx_E t) \) iff there exists an \( n \) such that \( s_n = t_n \).

Hint:
Since \( R \) is terminating, there exists a bound \( m \) such that \( s_i = t_i \) for all \( i \geq m \).