TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 13

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Exercise 63

a) Consider the following set of identities:

$$E := \{f(f(x, y), z) \approx f(x, f(y, z)),\ f(x, x) \approx x,\ f(f(x, y), x) \approx x\}$$

Apply the rules of the improved completion procedure to E. Use a strategy that resembles the basic completion procedure, but simplifies rules as follows: upon adding new rules, simplify old ones by means of L-SIMPLIFY-RULE and R-SIMPLIFY-RULE.

Consider the proof

$$P := \langle f(x, f(y, f(y, x))),\quad f(x, f(f(y, y), x)),\quad f(x, f(y, x)),\quad f(f(x, y), x),\quad x\rangle\,.$$

Construct a rewrite proof P' in R_ω with $P \succ_{\mathcal{C}} P'$ using the proof of Lemma 7.21.

b) Consider the following set of identities:

$$E := \{x + (y + z) \approx (x + y) + z,\ f(x) + f(y) \approx f(x + y)\}$$

Apply the completion procedure described above to input E and the polynomial order induced by

$$P_f(X) = X + 1,\ P_+(X, Y) = XY^2.$$

Exercise 64

The semi-decision procedure described in the proof of Theorem 7.22 of the lecture is rather inefficient: For the input $s \approx_E t$, all R_i-normal forms of s and t are computed in the ith iteration of the `repeat`-loop. Show that the following modification of the procedure still yields a semi-decision procedure for the word problem:

- Begin with $s_0 := s$ and $t_0 := t$.
- After the ith `repeat`-loop, compute *one arbitrary* R_i-normal form s_i of s_{i-1} and *one arbitrary* R_i-normal form t_i of t_{i-1}.
- Output ‘yes’ ($s \approx_E t$) iff there exists an n such that $s_n = t_n$.

Hint:
Show the following: (1) For all $n \geq 0$, it holds that $s \approx_E s_n$ and $t \approx_E t_n$; and (2) Since R_∞ is terminating, there exists an n such that $s_n = s_m$ and $t_n = t_m$ for all $m \geq n$.