TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Description Logics

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Exercise Sheet 2

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Exercise 2.1 Let $\otimes$ be a continuous t-norm. Prove the following two statements:

(a) For every $x, y \in [0,1]$ the set $\{z \in [0,1] \mid x \otimes z \leq y\}$ has a maximum.

(b) $x \Rightarrow y = \max\{z \in [0,1] \mid x \otimes z \leq y\}$ satisfies

$$z \leq x \Rightarrow y \text{ iff } x \otimes z \leq y.$$

Exercise 2.2 Show that the following three binary operators are continuous t-norms:

Łukasiewicz t-norm: $x \otimes y = \max\{x + y - 1, 0\}$,

Product t-norm: $x \otimes y = x \cdot y$,

Gödel t-norm: $x \otimes y = \min\{x, y\}$.

and that their residua are

Łukasiewicz: $x \Rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ 1 - x + y & \text{otherwise} \end{cases}$

Product: $x \Rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ \frac{y}{x} & \text{otherwise} \end{cases}$

Gödel: $x \Rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ y & \text{otherwise} \end{cases}$

Exercise 2.3 A partial order on the set of all t-norms can be defined naturally as follows. Let $\otimes_1$ and $\otimes_2$ denote two t-norms. We write

$$\otimes_1 \leq \otimes_2 :\Leftrightarrow \forall u, v \in [0,1] : u \otimes_1 v \leq u \otimes_2 v.$$

Find two t-norms $\otimes_{\text{min}}$ and $\otimes_{\text{max}}$ such that every t-norm $\otimes$ satisfies $\otimes_{\text{min}} \leq \otimes \leq \otimes_{\text{max}}$.