



## Fuzzy Description Logics

Summer Semester 2017

### Exercise Sheet 3

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**Exercise 3.1** Prove Lemma 2.5 from the lecture: if  $\otimes$  is the Łukasiewicz t-norm, then the following equivalences hold for every  $x, y \in [0, 1]$

- (a)  $\ominus \ominus x = x$ ,
- (b)  $x \Rightarrow y = \ominus x \oplus y$ ,
- (c)  $x \oplus y = \ominus(\ominus x \otimes \ominus y)$ .

**Exercise 3.2** Using ordinal sums, construct a continuous t-norm where exactly 3 values from  $[0, 1]$  are idempotent, i.e. exactly 3 values satisfy  $x \otimes x = x$ .

**Exercise 3.3** Consider the fuzzy ABox

$$\mathcal{A} = \{\langle A(a) \geq 0.5 \rangle, \langle r(a, b) \geq 0.9 \rangle, \langle r(a, c) \geq 0.7 \rangle\}$$

and the fuzzy TBox

$$\mathcal{T} = \{\langle A \sqsubseteq \forall r. (A \sqcup \neg B) \geq 0.8 \rangle, \\ \langle A \sqsubseteq \exists r. \neg B \geq 0.9 \rangle\}$$

where the Gödel t-norm is used. Present a model of  $\mathcal{A}$  and  $\mathcal{T}$  that is not crisp.

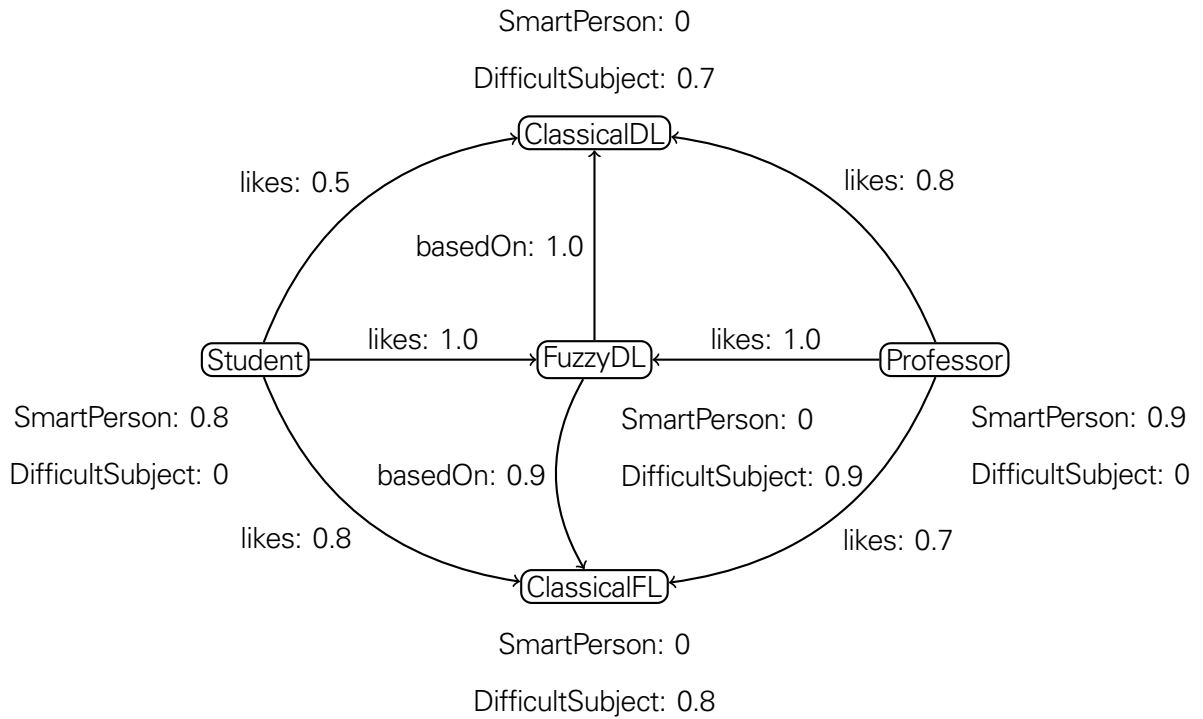
**Exercise 3.4** Consider a set of concept names

$$\mathcal{N}_C = \{\text{DifficultSubject}, \text{SmartPerson}\}$$

and a set of role names

$$\mathcal{N}_R = \{\text{likes}, \text{basedOn}\}.$$

Let a fuzzy interpretation be given by the following graphical representation (absent edges are meant to be read as 0, e.g. in this example  $\text{likes}^{\mathcal{I}}(\text{Student}, \text{Professor}) = 0$ ).



For the Gödel t-norm, give the interpretations of the following concept descriptions.

- $\text{SmartPerson} \sqcap \exists \text{likes}.\text{DifficultSubject}$
- $\forall \text{likes}.\exists \text{basedOn}.\text{DifficultSubject}$
- $\exists \text{likes}.\forall \text{basedOn}.\text{DifficultSubject}$