Fuzzy Description Logics

Exercise Sheet 4

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Exercise 4.1 Prove or disprove. For any t-norm \( \otimes \) the following equivalences hold:

- \( \neg (C \sqcup D) \equiv \neg C \sqcap \neg D \)
- \( \neg (C \sqcap D) \equiv \neg C \sqcup \neg D \)

Exercise 4.2 Let \( A \) be a concept name. Construct a Lukasiewicz-\( \mathcal{ALC} \) ontology such that \( A^I(x) \in \{0.25, 0.75\} \) for every model \( I \) and \( x \in \Delta^I \).

Exercise 4.3 For which of the three standard t-norms \( \otimes \) are the following \( \otimes-\mathcal{ALC} \) ABoxes consistent?

- \( A_1 = \{ \langle A(a), 0.5 \rangle, \langle \neg (A \sqcap A), 1 \rangle \} \)
- \( A_2 = \{ \langle \forall r. A(a), 1 \rangle, \langle \exists r. \neg A(a), 0.1 \rangle \} \)

Exercise 4.4 Prove the missing cases of Lemma 3.4 from the lecture: For all complex concepts \( C \) and \( x \in \Delta^I, C^J(x) = 1(C^I(x)) \).

Exercise 4.5 Does Lemma 3.5 from the lecture hold for assertions of the form \( \langle C(a) \leq q \rangle \)? Explain why.