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3. Exercises for the Course „Automata and Logic“

Exercise 11:

Let Σ be an alphabet and $(M, \circ, 1)$ a monoid. Prove the following: every function $f : \Sigma \to M$ can be uniquely extended to a (monoid)-homomorphism $\Phi : \Sigma^* \to M$.

Exercise 12:

Let $\Sigma := \{a, b\}$, $M := \{0, 1, 2\}$, and define $\circ : M \times M \to M$ by setting $x \circ y := (x + y) \mod 3$. We define mappings $\Phi, \Phi' : \Sigma^* \to M$ by putting $\Phi(w) := |w| \mod 3$, and $\Phi(w) := |w|_a \mod 3$, where $|w|$ denotes the length of the word w and $|w|_a$ denotes the number of times that "a" occurs in w.

(a) Show that both Φ and Φ' are homomorphisms from the monoid $(\Sigma^*, \cdot, \varepsilon)$ into the monoid $(M, \circ, 0)$.

(b) For each of the three languages $\Phi^{-1}(\{0, 2\})$, $\Phi^{-1}(\{1\})$, and $(\Phi')^{-1}(\{1\})$, devise a finite automaton recognizing the language.

Exercise 13:

For a language $L \subseteq \Sigma^*$, we use $\overline{L}$ to denote the complement of L, i.e. the language $\Sigma^* \setminus L$. Let Σ be an alphabet, $L \subseteq \Sigma^*$ a language, and $(M, \circ, 1)$ a monoid. Prove that, if L is accepted by $(M, \circ, 1)$, then $\overline{L}$ is also accepted by $(M, \circ, 1)$.

Exercise 14:

Determine the syntactic monoid of the language a^*ba^*.

Exercise 15:

Let $L \subseteq \Sigma^*$ and $\approx$ be an equivalence relation on Σ^*. Consider the following property:

$$\text{for all } u, v \in \Sigma^*, \text{ if } u \in L \text{ and } u \approx v, \text{ then } v \in L. \qquad (*)$$

(a) The proof of Corollary 1.13 from the lecture depends on the fact that the syntactical congruence $\sim_L$ has Property $(*)$ hat. Prove this.

(b) Show that $\sim_L$ is the coarsest congruence relation with Property $(*)$.

(c) Show that the Nerode right congruence ρ_L is the coarsest right congruence with Property $(*)$.

Note: An equivalence relation $\approx_2$ is called *coarser* than an equivalence relation $\approx_1$ if, for all x, y, $x \approx_1 y$ implies $x \approx_2 y$ (note that, in this case, $\approx_2$ has *at most* as many equivalence classes as $\approx_1$).