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# 5. Exercises for the Course „Automata and Logic“

**Exercise 21:**

Let $(M, \circ, 1)$ be a monoid with two right absorbing elements, i.e. we have $M := \{1, a, b\}$ with operation $\circ$ defined as

| $\circ$ | $1$ | $a$ | $b$ |
|---|---|---|---|
| $1$ | $1$ | $a$ | $b$ |
| $a$ | $a$ | $a$ | $b$ |
| $b$ | $b$ | $a$ | $b$ |

Prove that there exists a regular language $L$ whose syntactic monoid is isomorphic to $(M, \circ, 1)$.

**Exercise 22:**

Let $V$ be the M-variety of all commutative finite groups. Show that, for the language

$$L = \{a^n \mid n \text{ is even}\},$$

it holds that $L \in L(V)_{\{a\}}$, but $L \notin L(V)_{\{a,b\}}$.

**Exercise 23:**

Determine a language $L \subseteq \{a, b\}^*$ such that the syntactic semi-group $S_L$ and the syntactic monoid $M_L$ are isomorphic.

**Exercise 24:**

For each of the following words over the alphabet $\{0,1\}^k$, give a corresponding interpretation over the predicate symbols $P_1, \ldots, P_k$ as discussed in the lecture:

for $k = 2$: $(1,1), (1,1), (1,1), (0,0)$
for $k = 3$: $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$
for $k = 3$: $(1,1,0), (1,0,1), (0,1,0), (1,1,1)$

Describe all interpretations that correspond to words of the language $L(((0,1) \cdot (1,0))^+) \subseteq (\{0,1\}^2)^+$.

**Exercise 25:**

Let $\Sigma := \{a, b\}$. For each of the following regular expressions $r_i$, give a first-order formula $\varphi_i$ such that $L(r_i) = L(\varphi_i)$:

(a) $r_1 = \Sigma^*$;

(b) $r_2 = \{\epsilon\}$;

(c) $r_3 = L((abb^*)^*)$;

(d) $r_4 = L(a^*b^* + b^*a^*)$;

(e) $r_5 = L((aaa \cdot \Sigma^*) + b^*)$.

For each of the following formulas $\varphi_i$, give a regular expression $r_i$ such that $L(\varphi_i) = L(r_i)$:

(a) $\varphi_1 = \exists x, y, z \,.\, x < y \wedge y < z$;

(b) $\varphi_2 = Q_b(\min)$
$\wedge \forall x \,.\, \big(Q_b(x) \to (\exists y \,.\, \mathrm{succ}(x,y) \wedge Q_a(y))\big)$
$\wedge \forall x \,.\, \big(Q_a(x) \to (\forall y \,.\, \mathrm{succ}(x,y) \to Q_b(y))\big)$;

(c) $\varphi_3 = \forall x \,.\, \big(Q_a(x) \to \exists y \,.\, (x < y \wedge Q_b(y))\big)$;

(d) $\varphi_4 = \forall x \,.\, \big(\max(x)$
$\vee \exists y \,.\, \mathrm{succ}(x,y) \wedge \max(y)$
$\vee Q_b(x) \to \forall y . y < x \to Q_b(y)\big)$.

(e) $\varphi_5 = \forall x \,.\, \big(Q_a \to \exists y \,.\, \mathrm{succ}(x,y) \wedge Q_b(y))\big)$
$\wedge \forall x \,.\, \big(Q_b \to \exists y \,.\, \mathrm{succ}(x,y) \wedge Q_a(y))\big)$