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7. Exercises for the Course „Automata and Logic"

Exercise 31:

Let $\Sigma := \{a, b, c, d\}$.

(a) For $L \subseteq \Sigma^*$ with

$$L := \{w \in \Sigma^* \mid w \text{ starts with } a \text{ or } b\} \cap \{w \in \Sigma^* \mid |w| \geq 3 \text{ and } w \text{ starts and ends with the same symbol}\},$$

give a quantifier-free formula φ with $L(\varphi) = L$ using the signature $\{Q_a, Q_b, Q_c, Q_d, <, \mathsf{min}, \mathsf{max}, \mathsf{s}, \mathsf{p}\}$.

(b) Let

$$\varphi := \neg(\neg Q_a(s(s(p(s(\min))))) \vee (s(\min) < p(p(\max)))).$$

Use the method described in the proof of Theorem 2.11 to describe $L(\varphi)$ as a Boolean combination of languages $\{w\Sigma^* \mid w \in \Sigma^*\} \cup \{\Sigma^* w \mid w \in \Sigma^*\}$.

Exercise 32:

Let Σ be an alphabet. A language $L \subseteq \Sigma^*$ is called *definite* for Σ if there exists an $n \in \mathbb{N}$ such that, for all $w \in L$, the following holds:

if $w = uv$ with $|u| = n$, then $u\Sigma^* \subseteq L$.

Analogous to Lemma 2.2 from the lecture, show that $L \subseteq \Sigma^*$ is definite for Σ if and only if L is a Boolean combination of languages of the form $\{w\Sigma^* \mid w \in \Sigma^*\}$.

Exercise 33:

Let $\Sigma = \{0, 1\}^k$. Using the result proved in Exercise 32, show that the following equivalence holds:

L is definite for Σ $\iff$ there exists a quantifier-free and closed first order formula φ over the signature $\{P_1, \ldots, P_k, <, \mathsf{min}, \mathsf{s}\}$ such that $L(\varphi) = L \setminus \{\varepsilon\}$.

Exercise 34:

Let Σ and Γ be alphabets and $L \subseteq \Sigma^*$. Prove or refute the following implications:

(a) $L \in \mathbf{SF}_\Sigma \implies L \in \mathbf{SF}_{\Sigma\cup\Gamma}$

(b) $L \in \mathbf{SF}_{\Sigma\cup\Gamma} \implies L \in \mathbf{SF}_\Sigma$

Exercise 35:

For $\Sigma := \{a, b\}$, check whether the following languages are star-free:

(a) $L := (ab)^*$

(b) $L := \{w \mid \text{the number of } a\text{s in } w \text{ is a multiple of } 3\}$

(c) $L := a(aba)^*b$

Use either the Theorem of Schützenberger (Theorem 3.6 from the lecture) or give a star-free description of the language in question.