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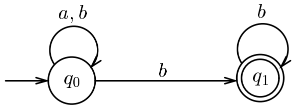
12. Exercises for the Course „Automata and Logic“

Exercise 50:

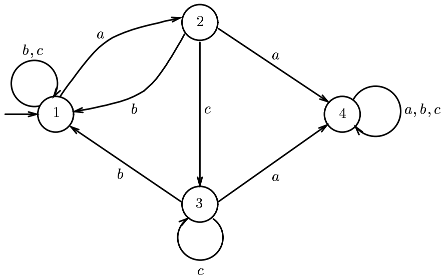
Let $\Sigma := \{a, b\}$ and $L \subseteq \Sigma^\omega$ be the ω-language recognized by the following Büchi-automaton:



Use the method presented in the lecture to construct a Büchi-automaton that recognizes the language $\Sigma^\omega \setminus L$.

Exercise 51:

Let $\Sigma := \{a, b, c\}$. From the transition system



we derive four Muller-automata $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$ and $\mathcal{A}_4$ by selecting the sets $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$, $\mathcal{F}_4$ of set of final states as follows:

- $\mathcal{F}_1 := \{\{1, 4\}, \{4\}\}$,
- $\mathcal{F}_2 := \{\{1, 2\}, \{3\}\}$,
- $\mathcal{F}_3 := \{\{1, 2, 3\}\}$ and
- $\mathcal{F}_4 := \{\{1\}, \{1, 2\}, \{3\}, \{1, 2, 3\}\}$.

Determine the ω-languages $L_\omega(\mathcal{A}_1)$, $L_\omega(\mathcal{A}_2)$, $L_\omega(\mathcal{A}_3)$ and $L_\omega(\mathcal{A}_4)$.

Exercise 52:

Let $\mathcal{A} := (Q, \Sigma, I, \Delta, F)$ be a Büchi-automaton and $\alpha \in \Sigma^\omega$.

- For an infinite path $p := q_0 \xrightarrow[\mathcal{A}]{a_0} q_1 \xrightarrow[\mathcal{A}]{a_1} q_2 \xrightarrow[\mathcal{A}]{a_2} \ldots$, let $\inf(p)$ denote the set of all states that are visited infinitely often on this path.
- The word $\alpha = a_0 a_1 a_2 \ldots$ is *strictly accepted* by $\mathcal{A}$ if there exists an infinite path $p := q_0 \xrightarrow[\mathcal{A}]{a_0} q_1 \xrightarrow[\mathcal{A}]{a_1} q_2 \xrightarrow[\mathcal{A}]{a_2} \ldots$ in $\mathcal{A}$ such that $\inf(p) = F$.
- An ω-language L is called *strictly recognizable* if there exists a Büchi-automaton $\mathcal{A}$ such that the words strictly accepted by $\mathcal{A}$ are precisely the words from L.

(a) Let Σ be an alphabet and $L = ab^\omega \cup ba^\omega$. Prove that L is Büchi-recognizable but not strictly recognizable.

(b) Is every strictly recognizable language Büchi-recognizable? Explain your answer.

Exercise 53:

For each of the following languages L_i, give an S1S-formula φ_i such that $L_\omega(\varphi_i) = L_i$:

(a) $L_1 = (abb^*)^\omega$;

(b) $L_2 = ((aa)^+(bb)^+)^\omega$;

(c) $L_3 = (aaa)^+ b(a + b)^\omega$.

Exercise 54:

Let $L := (a^+b)^\omega \cup (b^+a)^\omega$. **Use the proof of Theorem 5.4** to construct a closed S1S-formula φ with $L_\omega(\varphi) = L$.