13. Exercises for the Course
„Logic-based Knowledge Representation“

Exercise 43:
Prove that the following $K_n$-formulas are theorems:
(a) $[n]([m](p \lor q) \lor \neg [m](p \lor q))$
(b) $[m](p \land q) \rightarrow [m]q$ and $[m](p \land q) \rightarrow [m]p$
(c) $[m](p \land q) \rightarrow ([m]p \land [m]q)$

Exercise 44:
Let $\Delta$ and $\Gamma$ be sets of $K_n$-formulas. Prove or refute the following claims:
(a) if $\Gamma$ is consistent and $\Delta \subseteq \Gamma$, then $\Delta$ is consistent.
(b) if $\Gamma$ is inconsistent and $\Gamma \subseteq \Delta$, then $\Delta$ is inconsistent.
(c) Let $\Gamma = \bigcup_{i \geq 0} \Gamma_i$. Then $\Gamma$ is consistent iff all $\Gamma_i$ are consistent.
(d) Let $\Gamma = \bigcup_{i \geq 0} \Gamma_i$ and $\Gamma_0 \subseteq \Gamma_1 \subseteq \Gamma_2 \cdots$. Then $\Gamma$ is consistent iff all $\Gamma_i$ are consistent.

Exercise 45:
Prove Lemma 7.16 from the lecture:
Lemma 7.16 Let $R_m$ be a reachability relation.
(a) If $R_m$ is symmetric and transitiv, then $R_m$ is euclidian.
(b) The following three conditions are equivalent:
   (i) $R_m$ is symmetric, transitiv, and seriell.
   (ii) $R_m$ is reflexiv and euclidian.
   (iii) $R_m$ is an equivalence relation, i.e., reflexiv, transitiv, and symmetric.

Exercise 46:
Show that the sets of theorems of $K_n$, $S4_n$, and $S5_n$ are recursively enumerable (in other words, partially decidable).