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14. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 47:

Let $\mathcal{K}^{\mathsf{T}_n}$ be the T_n-canonical Kripke structure and let Γ be the set of all formulas that are valid in all Kripke structures with reflexive accessibility relations.

Complete the proof of Theorem 7.18.1 by proving the following claim:

The fact that in $\mathcal{K}^{\mathsf{T}_n}$ all accessibility relations are reflexive implies that every formula $\varphi \in \Gamma$ is deducible in T_n. (You are asked to repeat the completeness argument from Theorem 7.11 for $\mathcal{K}^{\mathsf{T}_n}$ assuming without proof Lemma 7.13 and modifying Definition 7.12.)

Exercise 48:

A Kripke structure with set of worlds W and a single accessibility relation R_m is *universal* iff $R_m = W \times W$. Let Γ be the set of modal formulas valid in all universal Kripke structures.

Prove that $\Gamma = \{\varphi \mid \ \vdash_{\mathsf{S5}_n} \varphi\}$.

Exercise 49:

Let $\mathcal{K}$ be a Kripke structure with accessibility relation R_m, φ a modal logic formula, and F the axiom schema

$$(\neg[m]\neg\varphi) \Rightarrow [m]\varphi.$$

Prove the following claims:

(a) If R_m is a partial function, then F holds in $\mathcal{K}$, i.e., $\mathcal{K}, w \models (\neg[m]\neg\varphi) \Rightarrow [m]\varphi$ for all formulae φ and all worlds w in $\mathcal{K}$.

(b) If a formula ψ holds in all Kripke structures whose accessibility relation R_m is a partial function, then ψ can be deduced in $\mathsf{K}_n + \mathsf{F}$.