15. Exercises for the Course
„Logic-based Knowledge Representation“

Exercise 50:

(a) Let \( D \) := \( \{ \text{Italian}(x) : \text{lovesWine}(x), \frac{\text{French}(x) : \text{lovesWine}(x)}{\text{lovesWine}(x)} \} \)
W := \{ \text{Italian}(\text{Tom}) \lor \text{French}(\text{Tom}) \}.

Compute the extensions of \( D \) and \( W \) and decide whether Tom loves wine.

(b) Let \( D \) := \( \{ \frac{\text{true} : \text{usable}(x) \land \neg \text{broken}(x)}{\text{usable}(x)} \} \)
W := \{ \text{broken}(\text{leftArm}) \lor \text{broken}(\text{rightArm}) \}.

Compute the extensions of \( D \) and \( W \) and decide whether both arms are usable.

Exercise 51:

Consider the default theory \((D, W)\) from Example 8.5 of the lecture: for closed atomic formulae \(a, b, c\) of first order predicate logic, we define
\[
D := \{ \frac{a:b, c:\neg b}{\neg b} \}
\]
W := \{ a \}.

Prove that \((D, W)\) has no extension.

Exercise 52:

Let \((D, W)\) be a default theory.

- Let \( \mathcal{E}_{(D, W)} \) be the set of all extensions of \((D, W)\).
- We call \( E, E' \in \mathcal{E}_{(D, W)} \) incomparable, if neither \( E \subseteq E' \) nor \( E' \subseteq E \).

Prove or disprove the following claims:

(a) There exists a default theory \((D, W)\) with \(|\mathcal{E}_{(D, W)}| \geq 2\) and where all \( E, E' \in \mathcal{E}_{(D, W)} \) with \( E \neq E' \) are incomparable.

(b) There exists a default theory \((D, W)\) with \(|\mathcal{E}_{(D, W)}| \geq 2\) and where, for all \( E, E' \in \mathcal{E}_{(D, W)}\), either \( E \subseteq E' \) or \( E' \subseteq E \).

Exercise 53:

Let \( \Pi \) be a ground logic program without NAF. Prove that, if \( \Gamma \) is the (non-empty) set of all models of \( \Pi \), then \( \cap \Gamma \) is also a model of \( \Pi \).