

### 3. Exercises for the Course „Logic-based Knowledge Representation“

**Exercise 8:**

Define a generic frame that describes the prototypical object “computer science course”. Use slots

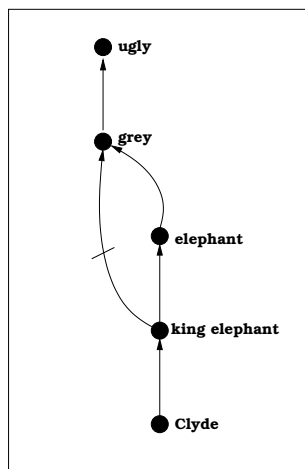
- *Title*,
- *Lecturer*,
- *Type of course*, and
- *Hours per week*.

Find other meaningful slots. Then construct an instance frame for the generic frame.

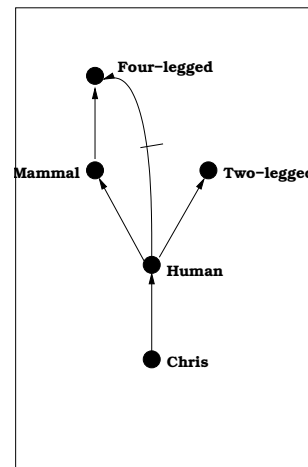
**Exercise 9:**

Consider the following non-monotonic inheritance networks and try to resolve the conflicts:

- (a) As is well known, king elephants are white rather than gray. Which color has Clyde?  
 (b) How many legs has Chris?



Network for Part (a)



Network for Part (b)

**Exercise 10:**

Let  $S$  be a set of propositional formulae in negation normal form (see Exercise 6).  $S$  contains a *clash* if  $\{\alpha, \neg\alpha\} \subseteq S$  for some propositional variable  $\alpha$ . If  $S$  does not contain a clash,  $S$  is *clash-free*. Next, we define the *tableau rules*:

- $\wedge$ -rule: If  $\beta_1 \wedge \beta_2 \in S$  and not  $\{\beta_1, \beta_2\} \subseteq S$ , then  $S := S \cup \{\beta_1, \beta_2\}$ .
- $\vee$ -rule: If  $\beta_1 \vee \beta_2 \in S$  and  $\{\beta_1, \beta_2\} \cap S = \emptyset$ , then  $S := S \cup \{\beta\}$  for some  $\beta \in \{\beta_1, \beta_2\}$ .

Please note that the  $\vee$ -rule is non-deterministic.

- (a) Apply the tableau rules exhaustively to

$$\{(p \vee q) \wedge (\neg p \vee \neg q)\}$$

in such a way that it yields a clash-free set of formulae.

(b) Apply the tableau rules exhaustively to

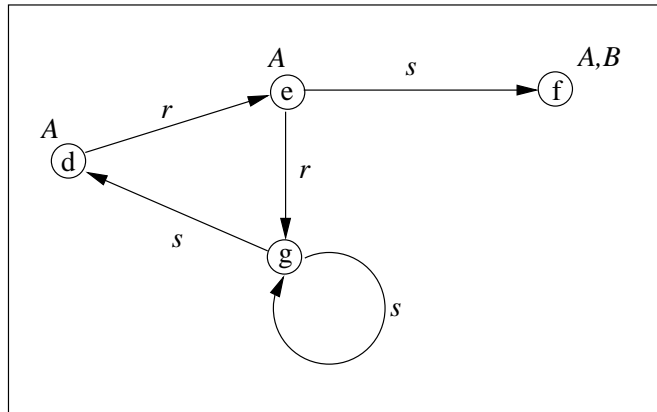
$$\{(p \vee q) \wedge (\neg p \vee \neg q)\}$$

in such a way that it yields set of formulae containing a clash.

- (c) Prove the following proposition: the tableau rules can be applied at most  $|S|$  times to a set of propositional formulae, where  $|S|$  denotes the sum of the length of all formulae in  $S$ .
- (d) Prove the following proposition: if the tableau can be applied exhaustively to  $S$  in such a way that the resulting set  $S'$  is clash-free, then  $S$  is satisfiable.
- (e) Prove that exhaustive application of tableau rules is a non-deterministic decision procedure for the satisfiability of propositional formulae.

**Exercise 11:**

Consider the (graphical representation of the) interpretation  $\mathcal{I}$  with  $\Delta^{\mathcal{I}} = \{d, e, f, g\}$ :



For each of the following concepts  $C$ , list all elements  $x$  of  $\Delta^{\mathcal{I}}$  such that  $x \in C^{\mathcal{I}}$ :

- $A \sqcup B$
- $\exists s. \neg A$
- $\forall s. A$
- $\exists s. \exists s. \exists s. \exists s. A$
- $\forall t. A \sqcap \forall t. \neg A$
- $\neg \exists r. (\neg A \sqcap \neg B)$
- $\exists s. (A \sqcap \forall S. \neg B) \sqcap \neg \forall r. \exists r. (A \sqcup \neg A)$