4. Exercises for the Course
„Logic-based Knowledge Representation“

Exercise 12:
Let Man, Woman, Male, Female, Human be concept names, and let has-child, is-brother-of, is-sister-of, is-married-to be roles names.

(a) Construct a TBox in which the concepts Mother, Father, Grandmother, Grandfather, Aunt, Uncle, Niece, Nephew, Mother of at least three sons, Father of at most two daughters are defined.

(b) Expand three of the definitions of your answer to part (a).

Exercise 13:
Prove that the procedure for expanding TBoxes of Theorem 3.6 always terminates, and that it returns a TBox that is equivalent to its input.

Hint: Count, for each concept name A, the number of concept names (directly or indirectly) used in the definition of A.

Exercise 14:
Let A be the ABox consisting of the following assertions:

(Ralf, Claudia) : likes
Claudia : blond
(Claudia, Jörg) : is-neighbour-of
(Jörg, Andrea) : is-neighbour-of

(a) Is A consistent?

(b) Is Ralf an instance of the concept ∃likes.(blond ∩ ∃is-neighbour-of.¬blond) with respect to A?

(c) Is Ralf an instance of the concept ∃likes.(∃is-neighbour-of. (∃is-neighbour-of. ¬blond)) with respect to A?

Exercise 15:
Which of the following statements are valid? Please give reasons for your answers.

(a) ∀r.(C ∩ D) ⊆ ∀r.C ∩ ∀r.D
(b) ∀r.C ∩ ∀r.D ⊆ ∀r.(C ∩ D)
(c) ∀r.(C ∪ D) ⊆ ∀r.C ∪ ∀r.D
(d) ∀r.C ∪ ∀r.D ⊆ ∀r.(C ∪ D)
(e) ∃r.(C ∩ D) ⊆ ∃r.C ∩ ∃r.D
(f) ∃r.C ∩ ∃r.D ⊆ ∃r.(C ∩ D)
(g) ∃r.C ∪ ∃r.D ⊆ ∃r.(C ∪ D)
(h) ∃r.(C ∪ D) ⊆ ∃r.C ∪ ∃r.D
Exercise 16:

It is easy to see that we can view description logic interpretations as structures in a first-order sense for signatures that contain only unary and binary predicate symbols.

Let \( \varphi(x) \) be a first-order predicate logic formula with one free variable \( x \), and let \( \mathcal{I} \) be an interpretation with domain \( \Delta^\mathcal{I} \) and \( d \in \Delta^\mathcal{I} \). We write \( \mathcal{I} \models \varphi(d) \) if \( \varphi(d) \) holds in \( \mathcal{I} \) (i.e., if the valuation that maps \( x \) to \( d \) makes \( \varphi(x) \) true in \( \mathcal{I} \)).

(a) Translate the concept \( \forall \forall. \forall.s.C \) into a first-order predicate logic formula \( \varphi(x) \) with one free variable \( x \) such that, in all interpretations \( \mathcal{I} \):

\[
(\forall \forall. \forall.s.C)^\mathcal{I} = \{d \in \Delta^\mathcal{I} \mid \mathcal{I} \models \varphi(d)\}.
\]

Try to use as few variables as possible.

(b) Let \( \mathcal{ALC} \) be the set of those \( \mathcal{ALCN} \)-concepts that do not use number restrictions. Define a translation function \( f \) that maps each \( \mathcal{ALC} \)-concept \( C \) into a first-order predicate logic formula \( \varphi_C(x) \) such that, in all interpretations \( \mathcal{I} \):

\[
C^\mathcal{I} = \{d \in \Delta^\mathcal{I} \mid \mathcal{I} \models \varphi_C(d)\}.
\]

Try to use as few variables as possible.

(c) Extend the translation function \( f \) from Part b to full \( \mathcal{ALCN} \)-concepts.