Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

5. Exercises for the Course „Logic-based Knowledge Representation"

Exercise 17:

Show that the 5 rules that are used in addition to the three Boolean ones (see Exercise 10) to transform a concept into a concept in negation normal form are equivalence preserving, i.e., show that, for concepts C, roles r, non-negative integers n, and positive integers n', the following equivalences hold:

$$\begin{array}{rclrcl} \neg(\forall r.C) & \equiv & \exists r.(\neg C) & \neg(\exists r.C) & \equiv & \forall r.(\neg C) \\ \neg(\leq n\ r) & \equiv & (\geq n+1\ r) & \neg(\geq n'\ r) & \equiv & (\leq n'-1\ r) \\ & & & \neg(\geq 0\ r) & \equiv & \bot \end{array}$$

Exercise 18:

Let A, B, C, and D be concept names and r a role name.

(a) Use the tableau-based algorithm presented in the lecture to decide consistency of the following ABoxes. If an ABox is consistent, describe a canonical model.

(i) $\mathcal{A}_1 = \{\mathsf{a}: (\exists \mathsf{r}.(\mathsf{C} \sqcup \mathsf{D}) \ \sqcap\ \forall \mathsf{r}.(\neg \mathsf{C} \sqcap \neg \mathsf{D})) \ \sqcup\ (\exists \mathsf{r}.\mathsf{D} \ \sqcap\ \forall \mathsf{r}.\forall \mathsf{r}.\neg \mathsf{D})\}$

(ii) $\mathcal{A}_2 = \{\mathsf{a}: \exists \mathsf{r}.(\leq 1\ \mathsf{r}) \sqcap \exists \mathsf{r}.(\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B} \sqcap \forall \mathsf{r}.\mathsf{C}),\ \mathsf{a}: \forall \mathsf{r}.((\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\neg \mathsf{A}) \sqcup (\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B}))\}$

(b) Use the tableau-based algorithm to decide the following subsumption relationship.

$$\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B} \ \sqsubseteq^?\ \exists \mathsf{r}.(\mathsf{A} \sqcap \mathsf{B})$$

Exercise 19:

Complete the proof of Lemma 4.6 by showing that, if the application of the

(a) $\sqcap$–Rule,

(b) $\forall$–Rule, or

(c) $\geq$-Rule

to A yields $A_1, \ldots, A_n$, then A is consistent iff, for some $1 \leq i \leq n$, A_i is consistent.