2. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 4:
Consider the problem of checking satisfiability of propositional logic formulas and the following algorithm given a formula \( \varphi \) with propositional variables \( p_1, \ldots, p_n \), enumerate all possible valuations for \( p_1, \ldots, p_n \) and for each one check whether it makes \( \varphi \) true. Return “yes” if such a valuation is found and “no” otherwise. Answer the following questions:

- Is this algorithm sound, complete, terminating?
- Is it a decision procedure? Is it a semi-decision procedure?
- If a formula is of length \( n \), how many steps the algorithm will need to check its satisfiability?
- How much memory will it use?
- Is it an NP algorithm?
- How can you modify it to get a better complexity?

Exercise 5:
A propositional formula is in negation normal form (NNF) if it is built using \( \land, \lor, \lnot \) only, and if negation occurs only in front of propositional variables. Two propositional formulas \( \alpha \) and \( \beta \) are equivalent if \( \alpha \iff \beta \) is valid. Prove that each propositional formula \( \alpha \) can be transformed into an equivalent one in NNF by first rewriting \( \alpha \) into an equivalent formula \( \alpha' \) that uses \( \land, \lor, \lnot \) only, and then by “pushing negation towards” \( \beta \), i.e., by applying the following three rules exhaustively to all subformulas of \( \alpha' \):

\[
\begin{align*}
\lnot \lnot \beta & \iff \beta \\
\lnot (\beta_1 \lor \beta_2) & \iff \lnot \beta_1 \land \lnot \beta_2 \\
\lnot (\beta_1 \land \beta_2) & \iff \lnot \beta_1 \lor \lnot \beta_2
\end{align*}
\]

Exercise 6:
Consider the following semantic network:

(a) Which nodes are concepts, which objects?
(b) Describe some possible meanings of property edges.
(c) Which color has Ralf’s car?
(d) What are the commonalities between sports car and opera?

Exercise 7:
Construct a semantic network that (partially) describes a university. Use concepts such as professor, assistant, student, and relationships such as teaches, is employed by, has a classmate, attends, and performs. Add additional concepts and relationships.