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10. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 31:

To complete the proof of Corollary 6.2., show that the transformation of an $\mathcal{FL}_0$-concept into normal form can be computed in polynomial time.

Exercise 32:

For the following $\mathcal{FL}_0$-concepts, show that C is *not* subsumed by D by giving an interpretation $\mathcal{I}$ such that $C^{\mathcal{I}} \not\subseteq D^{\mathcal{I}}$:

- $C = \forall r.\forall s.(A \sqcap B) \sqcap \forall r.(A \sqcap \forall s.\forall t.A), \quad D = \forall r.(A \sqcap \forall s.(A \sqcap B \sqcap \forall t.A \sqcap \forall r.B))$
- $C = \forall r.(A \sqcap \forall s.B) \sqcap \forall r.(B \sqcap \forall s.\forall t.A), \quad D = \forall r.(A \sqcap B) \sqcap \forall s.\forall r.A \sqcap \forall r.\forall s.B \sqcap \forall r.(\forall s.B \sqcap \forall t.A)$

Exercise 33:

As discussed in the lecture, the co-NP-hardness of subsumption in $\mathcal{FL}_0$ w.r.t. acyclic TBoxes can be proved by reducing the containment problem of acyclic finite automata, which is well-known to be co-NP-complete. Prove that subsumption in $\mathcal{FL}_0$ w.r.t. acyclic TBoxes is in co-NP (thus co-NP-complete) by reversing this reduction: devise a polynomial reduction from the subsumption problem of $\mathcal{FL}_0$ w.r.t. acyclic TBoxes to containment of acyclic finite automata.

Hints:

- Devise a normal form $\forall \mathcal{A}_1.A_1 \sqcap \cdots \sqcap \forall \mathcal{A}_n.A_n$ for $\mathcal{FL}_0$-concepts that takes into account TBoxes. In the normal form, the $\mathcal{A}_i$ are acyclic finite automata (instead of sets of words).
- To generate the automata $\mathcal{A}_i$, translate the *defined* concept names from the TBox into automata states. Add additional states if necessary.
- Reduce the subsumption of concepts in the new normal form to the containment problem of acyclic finite automata (for each primitive concept name in the input, one containment problem has to be solved).

Exercise 34:

Convert the following $\mathcal{EL}$-concepts into description trees:

- $\top$
- A
- $A \sqcap \exists R.\exists S.\top$
- $\exists R.\exists S.A \sqcap \exists T.B \sqcap \exists T.A$
- $B \sqcap \exists R.(A \sqcap B \sqcap \exists S.(A \sqcap B) \sqcap \exists T.A)$

Convert the following description trees into $\mathcal{EL}$-concepts: