**Lehrstuhl für Automatentheorie**
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

# 11. Exercises for the Course „Logic-based Knowledge Representation“

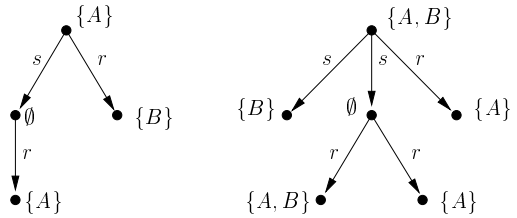
**Exercise 35:**

In the lecture, the translation of $\mathcal{EL}$-concepts into description trees has been presented on an intuitive level, only. Define this translation formally using induction on the structure of $\mathcal{EL}$-concepts.

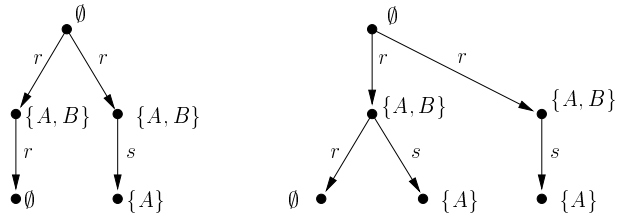
**Exercise 36:**

Compute all possible homomorphisms between the following description trees:

1.




2.




**Exercise 37:**

Let $\mathcal{L}$ be a Description Logic and $C$, $D$, and $E$ be $\mathcal{L}$-concept terms. Prove that the following properties hold:

(a) $\mathrm{lcs}(C, D) = \mathrm{lcs}(D, C)$

(b) $\mathrm{lcs}(C, \mathrm{lcs}(D, E)) = \mathrm{lcs}(\mathrm{lcs}(C, D), E)$

(c) The least common subsumer of $C$ and $D$ is unique up to equivalence.

**Exercise 38:**

Compute the least common subsumer of

(a) the following pairs of $\mathcal{FL}_0$-concept terms:

$$C = A_1 \sqcap A_2 \sqcap \forall r.(A_1 \sqcap \forall s.A_1 \sqcap \forall r.A_2) \sqcap \forall r.\forall s.\forall r.(A_1 \sqcap A_2 \sqcap \forall s.A_2)$$
$$D = A_2 \sqcap \forall r.\forall r.(A_2 \sqcap A_3) \sqcap \forall s.A_2 \sqcap \forall r.(A_2 \sqcap \forall s.(A_2 \sqcap \forall r.(A_2 \sqcap A_3)))$$

(b) the following pairs of $\mathcal{EL}$-concept terms:

$$C = A \sqcap \exists r.(A \sqcap \exists(A \sqcap B)) \sqcap \exists r.(B \sqcap \exists s.B) \sqcap \exists s.(A \sqcap \exists r.\top)$$
$$D = B \sqcap \exists r.(A \sqcap B \sqcap \exists s.(A \sqcap B \sqcap \exists s.(A \sqcap B))) \sqcap \exists s.(A \sqcap B) \sqcap \exists s.(B \sqcap \exists r.\top)$$

**Exercise 39:**

Consider the following $\mathcal{EL}$-concept terms:

$$C_1 = \exists r.(A \sqcap \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B))) \sqcap \exists r.(B \sqcap \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B)))$$
$$C_2 = \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap \exists r.(A \sqcap B)) \sqcap \exists r.(B \sqcap \exists r.(A \sqcap B)))$$
$$C_3 = \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B \sqcap \exists r.A \sqcap \exists r.B))$$

Compute $D = \mathrm{lcs}(C_1, \mathrm{lcs}(C_2, C_3))$ and prove that there is no $\mathcal{EL}$-concept term $E$ such that $D$ and $E$ are equivalent and $E$'s length is smaller than $2^4 - 1$ (i.e., the number of nodes in a tree of depth 3).