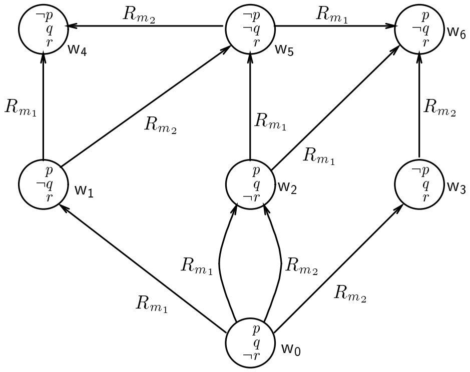
Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

12. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 40:

The Kripke structure $\mathcal{K} := (\{\mathsf{w}_0, \ldots, \mathsf{w}_6\}, \pi, R)$ is given by the following graph:



(a) Prove resp. disprove the following statements:

(i) $\mathcal{K}, \mathsf{w}_0 \models (r \to p) \wedge [m_1]\langle m_1 \rangle (p \to q)$

(ii) $\mathcal{K}, \mathsf{w}_1 \models \Big([m_2]\langle m_1 \rangle r \wedge \langle m_2 \rangle [m_1] r\Big) \to \Big([m_2]\langle m_2 \rangle q \vee [m_1]\langle m_1 \rangle \neg q\Big)$

(iii) $\mathcal{K}, \mathsf{w}_2 \models [m_1](p \to r) \wedge [m_2](p \wedge \neg p)$

(iv) $\mathcal{K}, \mathsf{w}_3 \models [m_1](p \vee q) \wedge \langle m_2 \rangle (p \to r) \wedge [m_2]\langle m_1 \rangle \neg q\,.$

(b) Give, for each formula in (a), the corresponding $\mathcal{ALC}$ concept term.

(c) Describe an interpretation $\mathcal{I}$ that corresponds one-to-one to the Kripke structure $\mathcal{K}$ (in the sense mentioned in the lecture).

Exercise 41:

For $n \in \mathbb{N} \setminus \{0\}$, let $M_n := \{m_1, \ldots, m_n\}$ be a set of modal parameters and $m \in M_n$. Prove resp. disprove the validity of the following formulae:

(a) $[m]p \to p$

(b) $[m]p \to [m][m]p$

(c) $p \to [m]\langle m \rangle p$

(d) $\langle m \rangle p \to [m]\langle m \rangle p$

(e) $([m](p \to q)) \to ([m]p \to [m]q)\,.$

Exercise 42:

In the lecture, the filtration technique was used to prove that K_n has the finite model property. Use the technique to prove that $\mathsf{K4}_n$ has the finite model property, where $\mathsf{K4}_n$ is syntactically the same logic as K_n, but the semantics is different: we only consider those Kripke structures where each accessibility relation is transitive.