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4. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 12:

Let Man, Woman, Male, Female, Human be concept names, and let has-child, is-brother-of, is-sister-of, is-married-to be roles names.

(a) Construct a TBox in which the concepts *Mother*, *Father*, *Grandmother*, *Grandfather*, *Aunt*, *Uncle*, *Niece*, *Nephew*, *Mother of at least three sons*, *Father of at most two daughters* are defined.

(b) Expand three of the definitions of your answer to part (a).

Exercise 13:

Prove that the procedure for expanding TBoxes of Theorem 3.6 always terminates, and that it returns a TBox that is equivalent to its input.
Hint: Count, for each concept name A, the number of concept names (directly or indirectly) used in the definition of A.

Exercise 14:

Let $\mathcal{A}$ be the ABox consisting of the following assertions:

$$\begin{array}{rcl} (\mathsf{Ralf},\mathsf{Claudia}):\mathsf{likes} & \quad & (\mathsf{Ralf},\mathsf{Jörg}):\mathsf{likes} \\ (\mathsf{Claudia},\mathsf{Jörg}):\mathsf{is\text{-}neighbour\text{-}of} & & (\mathsf{Jörg},\mathsf{Andrea}):\mathsf{is\text{-}neighbour\text{-}of} \\ \mathsf{Claudia}:\mathsf{blond} & & \mathsf{Andrea}:\neg\mathsf{blond} \end{array}$$

(a) Is $\mathcal{A}$ consistent?

(b) Is Ralf an instance of the concept $\exists\mathsf{likes}.(\mathsf{blond} \sqcap \exists\mathsf{is\text{-}neighbour\text{-}of}.\neg\mathsf{blond})$ with respect to $\mathcal{A}$?

(c) Is Ralf an instance of the concept $\exists\mathsf{likes}.(\exists\mathsf{is\text{-}neighbour\text{-}of}.\ (\forall\mathsf{is\text{-}neighbour\text{-}of}.\ \neg\mathsf{blond}))$ with respect to $\mathcal{A}$?

Exercise 15:

Which of the following statements are valid? Please give reasons for your answers.

$$\begin{array}{lrcl}
(a) & \forall r.(C \sqcap D) & \sqsubseteq & \forall r.C \sqcap \forall r.D \\
(b) & \forall r.C \sqcap \forall r.D & \sqsubseteq & \forall r.(C \sqcap D) \\
(c) & \forall r.(C \sqcup D) & \sqsubseteq & \forall r.C \sqcup \forall r.D \\
(d) & \forall r.C \sqcup \forall r.D & \sqsubseteq & \forall r.(C \sqcup D) \\
(e) & \exists r.(C \sqcap D) & \sqsubseteq & \exists r.C \sqcap \exists r.D \\
(f) & \exists r.C \sqcap \exists r.D & \sqsubseteq & \exists r.(C \sqcap D) \\
(g) & \exists r.C \sqcup \exists r.D & \sqsubseteq & \exists r.(C \sqcup D) \\
(h) & \exists r.(C \sqcup D) & \sqsubseteq & \exists r.C \sqcup \exists r.D
\end{array}$$

Exercise 16:

It is easy to see that we can view description logic interpretations as structures in a first-order sense for signatures that contain only unary and binary predicate symbols.

Let $\varphi(x)$ be a first-order predicate logic formula with one free variable x, and let $\mathcal{I}$ be an interpretation with domain $\Delta^{\mathcal{I}}$ and $d \in \Delta^{\mathcal{I}}$. We write $\mathcal{I} \models \varphi(d)$ if $\varphi(d)$ holds in $\mathcal{I}$ (i.e., if the valuation that maps x to d makes $\varphi(x)$ true in $\mathcal{I}$).

(a) Translate the concept $\forall r.\forall s.C$ into a first-order predicate logic formula $\varphi(x)$ with one free variable x such that, in all interpretations $\mathcal{I}$:

$$(\forall r.\forall s.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \mathcal{I} \models \varphi(d)\}.$$

Try to use as few variables as possible.

(b) Let $\mathcal{ALC}$ be the set of those $\mathcal{ALCN}$-concepts that do not use number restrictions. Define a translation function f that maps each $\mathcal{ALC}$-concept C into a first-order predicate logic formula $\varphi_C(x)$ such that, in all interpretations $\mathcal{I}$:

$$C^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \mathcal{I} \models \varphi_C(d)\}.$$

Try to use as few variables as possible.

(c) Extend the translation function f from Part b to full $\mathcal{ALCN}$-concepts.