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8. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 25:

Recall the following notions:

1. A *polynomial reduction* from a problem $P \subseteq X$ to a problem $P' \subseteq X'$ is a polynomial time computable function σ that takes every $x \in X$ to a $\sigma(x) \in X'$ such that $x \in P$ iff $\sigma(x) \in P'$.
2. A problem P is PSPACE-*hard* iff, for every problem P' that is *in* PSPACE, there exists a polynomial reduction σ from P' to P.

Prove the following claims:

(a) If a problem P_1 is PSPACE-hard and there exists a polynomial reduction from P_1 to another problem P_2, then P_2 is PSPACE-hard.

(b) If a problem P_1 is undecidable and there exists a polynomial reduction from P_1 to another problem P_2, then P_2 is undecidable.

Exercise 26:

To extend the $\mathcal{ALCN}$-tableau algorithm from the lecture such that it can, additionally, deal with role value maps, one could think of adding the following two rules:

Replace $\mathcal{A} \in \mathcal{M}$ by $\mathcal{A}'$ iff the following conditions hold:

1. $a : (R \sqsubseteq S) \in \mathcal{A}$, where $R = r_1 \cdots r_n$ and $S = s_1 \cdots s_m$
2. $\{(a, b_1) : r_1, (b_1, b_2) : r_2, \ldots, (b_{n-1}, b_n) : r_n\} \subseteq \mathcal{A}$
3. there exist no $c_1, \ldots, c_m$ such that $c_m = b_n$ and $\{(a, c_1) : s_1, (c_1, c_2) : s_2, \ldots, (c_{m-1}, c_m) : s_m\} \subseteq \mathcal{A}$
4. $\mathcal{A}' = \mathcal{A} \cup \{(a, c_1) : s_1, (c_1, c_2) : s_2, \ldots, (c_{m-1}, c_m) : s_m\}$ where $c_1, \ldots, c_{m-1}$ are new individuals and $c_m = b_n$.

Then do the following:

(a) Apply the extended tableau algorithm to the ABox

$$\{a : (\exists r.\exists s.\exists s.A) \sqcap (\forall r.\neg A) \sqcap (rs \sqsubseteq r)\}$$

and decide whether it is consistent.

(b) Devise an ABox on which the extended algorithm does not terminate.