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9. Exercises for the Course „Logic-based Knowledge Representation"

Exercise 27:

We define a slight variant of the domino problem: A solution of a domino system $\mathcal{D} = (D, H, V)$ (as defined in the lecture) *w.r.t. an initial condition* $a = a_0, \ldots, a_n$ is a mapping $\tau : \mathbb{N} \times \mathbb{N} \to D$ such that

- $\tau(x, y) = d$ and $\tau(x + 1, y) = d'$ implies $(d, d') \in H$,
- $\tau(x, y) = d$ and $\tau(x, y + 1) = d'$ implies $(d, d') \in V$, and
- $\tau(i, 0) = a_i$ for $i \leq n$.

Prove that it is undecidable whether a domino system $\mathcal{D}$ has a solution w.r.t. an initial condition a.

Hints:

(a) Reduce the halting problem of Turing machines (TMs) to this problem. Consider only TMs which use one-side infinite tapes and are started on the empty tape (for this class of TMs, the halting problem is already undecidable).

(b) Use the first row $\{(i, 0) \mid i \in \mathbb{N}\}$ of the $\mathbb{N} \times \mathbb{N}$ structure to represent the initial tape content, the second row $\{(i, 1) \mid i \in \mathbb{N}\}$ to represent the tape content after the first step of the TM, and so forth.

(c) Define additional tile types that, apart from determining the content of a tape cell, represent the position of the TM's head and its current state.

(d) Use the initial condition to ensure that the TM starts in the initial state and that the tape is initially empty.

(e) Be careful: the head's position and state must be unique for each step of the computation.

Exercise 28:

Let $\mathcal{D} = (D, H, V)$ be a domino system. A solution $\tau : \mathbb{N} \times \mathbb{N} \to D$ is called *periodic* iff there exist natural numbers $h, v > 0$ such that $\tau(x, y) = \tau(x + h, y) = \tau(x, y + v)$. Show that

(a) It is partially decidable whether a domino system has *no* solution.

(b) It is partially decidable whether a domino system has a *periodic* solution.

(c) There exist solvable domino systems that admit only non-periodic solutions.

Exercise 29:

Use a reduction of the domino problem to prove that satisfiability of first-oder logic formulas is undecidable. In the reduction, you are allowed to use equality, but no constants and function symbols.

Exercise 30:

For the following pairs of $\mathcal{FL}_0$-concepts C, D, use the algorithm from the lecture to decide whether C is subsumed by D:

(a) $C = \forall R.\forall R.(A \sqcap B), \quad D = \forall R.\forall R.A$

(b) $C = \forall R.A \sqcap \forall S.B, \quad D = \forall R.(A \sqcap B) \sqcap \forall S.(A \sqcap B)$

(c) $C = \forall R.(A \sqcap \forall S.B), \quad D = \forall R.(\forall S.B) \sqcap \forall R.A$

(d) $C = \forall R.\forall S.A \sqcap \forall S.\forall R.A, \quad D = \forall R.\forall R.A \sqcap \forall S.\forall S.A$

(e) $C = A \sqcap \forall R.A \sqcap \forall S.B, \quad D = A \sqcap \forall R.A$