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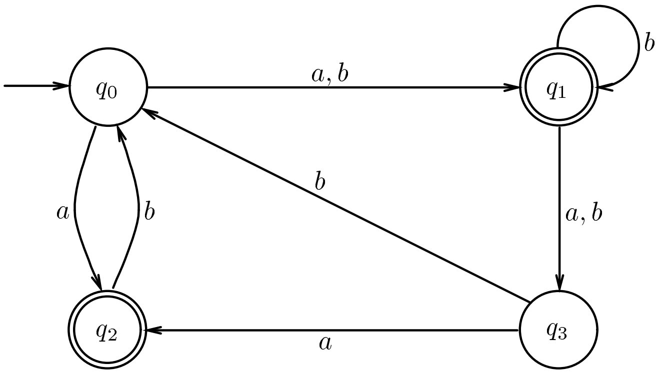
1. Exercises for the Course „Automata and Logic“

Exercise 1:

Let the nondeterministic finite automaton $\mathcal{A} := (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \{q_0\}, \Delta, \{q_1, q_2\})$ be given by the following transition system:



Apply the power set construction to $\mathcal{A}$ in order to construct a *deterministic* automaton that accepts the same language as $\mathcal{A}$.

Exercise 2:

Let $\Sigma := \{a, b\}$ be an alphabet and $\alpha := a^*b^* + b^*a^*$ a regular expression over Σ. Give a regular expression β for the complement language of α (i.e., a regular expression β describing the set of words over the alphabet Σ which are not contained in the language described by α).

Exercise 3:

Let Σ be a finite alphabet, $L \subseteq \Sigma^*$ a regular language over Σ, and $\sim_L \subseteq \Sigma^* \times \Sigma^*$ the *Nerode right congruence*, i.e. we have $u \sim_L v$ if and only if, for all $w \in \Sigma^*$, the following holds:

$$uw \in L \Longleftrightarrow vw \in L.$$

Let the deterministic finite automaton $\mathcal{A}_L := (Q_L, \Sigma, q_L, \delta_L, F_L)$ be defined as follows:

- $Q_L := \{[u] \mid u \in \Sigma^*\}$ where $[u] := \{v \in \Sigma^* \mid u \sim_L v\}$,
- $q_L := [\varepsilon]$ for the empty word ε,
- $\delta_L([u], a) := [ua]_{\sim_L}$ for $u \in \Sigma^*$ and $a \in \Sigma$,
- $F_L := \{[u]_{|} u \in L\}$.

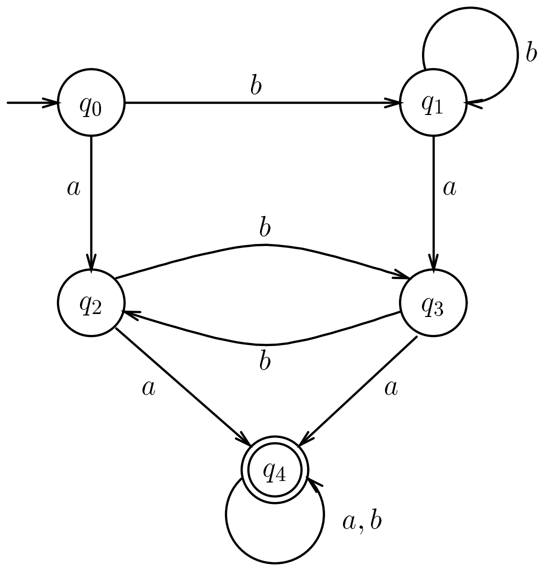
Show the following:

(a) $\mathcal{A}_L$ is well-defined.

(b) $\mathcal{A}_L$ is minimal, i.e.. for every deterministic finite automaton $\mathcal{A} := (Q, \Sigma, q_0, \delta, F)$ with $L(\mathcal{A}) = L$, we have $|Q_L| \leq |Q|$.

Exercise 4:

Let $\mathcal{A}$ be the finite automaton that accepts words over the alphabet $\Sigma := \{a, b\}$ and is described by the following transition system:



Construct an automaton $\mathcal{A}'$ such that $L(\mathcal{A}') = L(\mathcal{A})$ and $\mathcal{A}'$ is minimal. Hint: use the Nerode right congruence $\sim_{L(\mathcal{A})}$ and construct the automaton $\mathcal{A}_{L(\mathcal{A})}$ induced by the equivalence classes of $\sim_{L(\mathcal{A})}$.

Exercise 5:

Prove the following by giving a decision procedure:

(a) The *emptiness problem* for regular languages is decidable.

(b) The *inclusion problem* for regular languages is decidable.

Reminder:

- The *emptiness problem* for regular languages is defined as follows:

  Given: A regular language L (through a finite automaton or a regular expression).

  Question: Does $L = \emptyset$ hold?

- The *inclusion problem* for regular languages is defined as follows:

  Given: Two regular languages L_1 and L_2 (through finite automata or a regular expressions).

  Question: Does $L_1 \subseteq L_2$ hold?