Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

14. Exercises for the Course „Automata and Logic“

Exercise 61:

Let Σ be an alphabet with arity function and $x \in \Sigma_0$. Prove or refute that, for all tree languages $U, V, W \subseteq \mathbf{T}_\Sigma$, it holds that

$$(U \cdot^x V) \cdot^x W = U \cdot^x (V \cdot^x W)\,.$$

Exercise 62:

Construct regular expressions for the following tree languages:

(a) The language of all trees that represent arithmetic expressions over the binary symbols $+$ and $\cdot$, the unary symbol $-$, and the variables (i.e., nullary symbols) x, y, and z.

(b) The language of all trees that represent regular expressions over the alphabet $\Sigma_L := \{a, b\}$.

You may add auxiliary symbols to the used alphabets if necessary.

Exercise 63:

Let $\Sigma = \{f, n, a\}$ be an alphabet with arity function $\nu = \{f \mapsto 2, n \mapsto 1, a \mapsto 0\}$. Show that every *finite* tree language $L \subseteq T_\Sigma$ is a regular tree language.

Exercise 64:

Let $\Sigma = \{\vee, \wedge, \neg, \top, \bot\}$ be an alphabet with the obvious arity function. The following RL-tree automaton $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ accepts precisely those trees that represent Boolean expressions evaluating to “false”:

- $Q := \{0, 1\}$
- $I = \{0\}$
- $\Delta_\neg(0) = 1$, $\Delta_\neg(1) = 0$
- $\Delta_\wedge(0) = \{(0,0), (0,1), (1,0)\}$, $\Delta_\wedge(1) = \{(1,1)\}$
- $\Delta_\vee(0) = \{(0,0)\}$, $\Delta_\vee(1) = \{(0,1), (1,0), (1,1)\}$
- $F(\top) = 1$, $F(\bot) = 0$.

Use the method from the proof of Theorem 6.20 to construct a regular expression for the tree language recognized by $\mathcal{A}$.