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15. Exercises for the Course „Automata and Logic“

Exercise 65:

Let $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ be an RL-tree automaton. We inductively define sets of states $R_0, R_1, \ldots$ as follows:

- $R_0 := \bigcup_{a \in \Sigma_0} F(a)$;
- $R_{i+1} := R_i \cup \{q \in Q \mid \text{ there is } f \in \Sigma \text{ and } q_1, \ldots, q_k \in R_i \text{ such that } (q_1, \ldots, q_k) \in \Delta_f(q)\}$

Then set $R(\mathcal{A}) = \bigcup_{i \geq 0} R_i$.

(a) Compute the set R for the following tree automaton:

- $Q = \{1, \ldots, 4\}$;
- $\Sigma = \{g, n, a, b\}$ with $\nu(g) = 2$, $\nu(n) = 1$, and $\nu(a) = \nu(b) = 0$;
- $I = \{1\}$;
- $\Delta_n(1) = \{1\}$, $\Delta_n(2) = \{3\}$, $\Delta_n(3) = \{2, 1\}$, $\Delta_n(4) = \{3, 1\}$;
- $\Delta_g(1) = \{(1, 1), (1, 2), (3, 4), (4, 1)\}$;
- $\Delta_g(2) = \emptyset$; $\Delta_g(3) = Q \times Q$;
- $\Delta_g(4) = \{(1, 2), (1, 4), (2, 4), (1, 2)\}$;
- $F(a) = \{2\}$; $F(b) = \{2, 3\}$.

(b) Prove that, for any RL-tree automaton $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$, the following holds:

(i) $L(\mathcal{A}) = \emptyset$ if and only if $R(\mathcal{A}) \cap I = \emptyset$;

(ii) $R(\mathcal{A})$ can be computed in time polynomial in the size of Q.

Exercise 66:

Let $\Sigma := \{a, b\}$ be an alphabet with two binary symbols and

$$L := \{t \in \mathbf{T}_\Sigma^\omega \mid \text{there is a path in } t \text{ containing only finitely many } a\text{'s}\}.$$

(a) Is L Rabin-recognizable?

(b) Is L Büchi-recognizable?

Exercise 67:

Let Σ be an alphabet with arity function containing at least two binary symbols f and g. Prove or refute:

The ω-tree language $L := \{f(t, t) \mid t \in \mathbf{T}_\Sigma^\omega\}$ is Büchi-recognizable.

Exercise 68:

Complete the proof of Theorem 7.11 by showing the following:

Let Σ be an alphabet of binary symbols and $L \subseteq \mathbf{T}_\Sigma^\omega$ be such that there exists a set $F = \{f_1, \dots, f_m\}$ of nullary symbols and recognizable tree-languages $L_0, \dots, L_m \subseteq \mathbf{T}_{\Sigma \cup F}$ such that

$$L = L_0 \cdot^{(f_1,\dots,f_m)} (L_1, \dots, L_m)^{\omega,(f_1,\dots,f_m)}.$$

Then L is Büchi-recognizable.

Exercise 69:

Let Σ be an alphabet of binary symbols.

(1) For each ω-tree $t \in \mathbf{T}_\Sigma^\omega$, we define a language of ω-words $\mathbf{path}(t) \subseteq \Sigma^\omega$ as follows:[1]

$$\mathbf{path}(t) := \{\alpha \in \Sigma^\omega \mid \text{ there is a path } i_0 i_1 i_2 \cdots \text{in } t \text{ such that } t(i_0 \cdots i_n) = \alpha(n) \text{ for all } n \in \mathbb{N}\}$$

For $B \subseteq \mathbf{T}_\Sigma^\omega$, let $\mathbf{path}(B) := \bigcup_{t \in B} \mathbf{path}(t)$.

(2) For a language $L \subseteq \Sigma^\omega$ of ω-words, let $\mathbf{tree}(L) := \{t \in \mathbf{T}_\Sigma^\omega \mid \mathbf{path}(t) \subseteq L\}$.

Prove or refute:

(a) $\mathbf{tree}(\mathbf{path}(B)) = B$

(b) $\mathbf{path}(\mathbf{tree}(L)) = L$

(c) If B is Büchi-recognizable, then $\mathbf{path}(B)$ is also Büchi-recognizable.

(d) If $\mathbf{path}(B)$ is Büchi-recognizable, then B is also Büchi-recognizable.

(e) If L is Büchi-recognizable, then $\mathbf{tree}(L)$ is also Büchi-recognizable.

[1] We may consider ω-words over the alphabet Σ by ignoring the arity of the symbols in Σ.