Theorem Proving and Equality

Exercise 1 *Change into the prenex normal form and skolemize:*

1. $\forall x(Person(x) \rightarrow \exists y Parent(y, x))$
2. $\forall x_0(\forall x_1 R_1(x_1, x_0) \rightarrow \exists x_1 R_2(x_0, x_1))$

Exercise 2 *Put into skolemized clauses:*

1. $\exists x.(Dog(x) \wedge Owns(Jack, x))$
2. $\forall x.AnimalLover(x) \rightarrow (\exists y.Animal(y) \wedge Owns(x, y))$
3. $\forall x.(\exists y.Dog(y) \wedge Owns(x, y) \rightarrow AnimalLover(x))$
4. $\forall x.AnimalLover(x) \rightarrow (\forall y.Aminal(y) \rightarrow \neg Kills(x, y))$

Exercise 3 *Prove with the help of $\mathcal{B}$ that the following formula is a FOL-theorem:*

$\forall x[(P(x) \rightarrow Q(x)) \wedge (Q(x) \rightarrow R(x)) \wedge (P(x) \rightarrow \neg R(x))] \rightarrow \neg P(c)$