Theorem Proving and Equality

Exercise 37 *Prove Lemma 9:*
For every set of ground clauses S, the set R_S of rules generated for S is convergent. If $R_C^* \models C$, then $R^* \models C$.

Hint: look at the proofs of lemma 5 and 6.

Exercise 38 *Prove Lemma 10:*
If a clause C of the form $\Gamma \to l \approx r, \Delta$ generates the rule $l \to r$, then $R^* \models \Gamma$ and $R^* \not\models \Delta$.

Exercise 39 *Look at the slide 17. Notice small changes in the conditions for Superposition left, Equality Resolution and Equality Factoring concerning selection function.*

Complete and correct accordingly the completeness proof given in the lecture.

You have to complete the following cases:

3. $C\gamma$ is of the form $\Gamma\gamma \to s\gamma \approx t\gamma, \Delta\gamma$($s\gamma$ is the maximal term), $s\gamma \approx t\gamma$ is strictly maximal with respect to the equations in $\Delta\gamma$, nothing is selected in $C\gamma$, but $s\gamma$ is reducible by $R_{C\gamma}$.

4. $C\gamma$ is of the form $\Gamma\gamma \to s\gamma \approx t\gamma, \Delta\gamma$($s\gamma$ is the maximal term), $s\gamma \approx t\gamma$ is strictly maximal with respect to the equations in $\Delta\gamma$, nothing is selected in $C\gamma$, $s\gamma$ is irreducible by $R_{C\gamma}$, but $R_{C\gamma}^* \models t\gamma \approx t'\gamma$ for an equation $s\gamma \approx t'\gamma$ in $\Delta\gamma$.

5. $C\gamma = \Gamma\gamma, s\gamma \approx s'\gamma \to \Delta\gamma$, ($s\gamma = s'\gamma$), $s\gamma \approx s'\gamma$ is selected or nothing is selected in $C\gamma$ and $s\gamma \approx s'\gamma$ is maximal in $\Gamma\gamma \cup \Delta\gamma$.

6. $C\gamma = \Gamma\gamma, s\gamma \approx t\gamma \to \Delta\gamma$, $s\gamma \approx t\gamma$ is selected or nothing is selected in $C\gamma$ and $s\gamma \approx t\gamma$ is maximal in $\Gamma\gamma \cup \Delta\gamma$.

Exercise 40 *Given the order on ground terms: $a \succ b \succ c \succ d$ and ground clauses:*
$c \approx d \to$,
$\to b \approx d$,
$\to a \approx b, a \approx d$,
$a \approx d \to a \approx c$,

construct a set of rules R according to Definition 12.