Theorem Proving and Equality

Exercise 4 *Prove the following claim:*
if S is a set of clauses saturated by a set of inference rules Γ, and Γ is refutation complete, then the empty clause is in S or S is satisfiable.

Exercise 5 *Prove the following claim:*
If S is a finite set of clauses and S' is a saturation of S by the inferences from a finite set of inference rules Γ, and D is a clause in S', then D' has a finite derivation from S (i.e. there is a finite sequence of clauses $D_1, \ldots, D_n, D$, such that each clause in this sequance is either in S or is obtained form other clauses by one of the rules in Γ).

Exercise 6 *Write in full the proof of* **Fact 1***:*
A set of clauses S is satisfiable in FOLE iff $S \cup \mathcal{E}$ is satisfiable in FOL.

Exercise 7 *Use inferences in $\mathcal{B}$ (binary resolution and positive factoring) to prove that $a \approx c$ follows from $b \approx a$ and $b \approx c$.*