Theorem Proving and Equality

Exercise 11 *Let* $\Sigma = \{f^{(2)}, i^{(1)}, e^{(0)}\}$ *and* $i > f > e$. *Decide which is a bigger term with respect to* $>_{lpo}$*:*

1. $f(x, e)$, x *?*
2. $i(e)$, e*?*
3. $f(x, f(y, z))$, $f(f(x, y), z)$ *?*

Exercise 12 *Assume that* $>_{lpo}$ *is a reduction order, and use Fact 8 to prove that the term rewriting system* R_{Ack} *is terminating.*

R_{Ack} *is defined by the following rules:*

1. $a(0, y) \to s(y)$
2. $a(s(x), 0) \to a(x, s(0))$
3. $a(s(x), s(y)) \to a(x, a(s(x), y))$

Use the precedence: $a > s$.

Exercise 13 *Prove that if the precedence on the signature is total, then the lexicographic path order is total on the ground terms (the terms with no variables).*

Hint: use induction on the size of both terms to be compared, (i.e. if $|s|$ *and* $|t|$ *are sizes of the terms* s *and* t*, then the induction is on* $|s| + |t|$*).*