Theorem Proving and Equality

Exercise 17 Prove that: *If $\succ$ is total on a set of elements, then $\succ_{mul}$ is also total on finite multisets over this set.*

Exercise 18 *Let the order on the ground atoms be the following:*
$B_2 \succ A_2 \succ B_1 \succ A_1 \succ B_0 \succ A_0$ *and let* $N = \{A_0 \vee B_0, B_0 \vee A_1, \neg B_0 \vee A_1, \neg B_0 \vee A_2 \vee B_1, \neg B_0 \vee \neg A_2 \vee B_1, \neg B_1 \vee B_2\}$.

Saturate N with inferences from $\mathcal{B}$ and construct I_N.

Exercise 19 *Let the order on the ground atoms be: $B \succ A$. Construct I_N for N where*

1. $N = \{\neg A, A \vee B, \neg B\}$.
2. $N = \{A \vee B, \neg A \vee B, A \vee \neg B, \neg A \vee \neg B\}$

Is I_N a model for N? Saturate N with $\mathcal{B}$.

Exercise 20 *Prove the refutation soundness and completeness of the inference system based on one rule:*

Binary resolution with factoring:

$$\frac{C \vee A_1 \vee \cdots \vee A_n \quad \neg B \vee D}{(C \vee D)\sigma}$$

where $\sigma = mgu(A_1, \ldots, A_n, B)$.