Theorem Proving and Equality

Exercise 21 *Assuming that we have an order on the ground clauses, we have extended it to the non-ground clauses as follows:* $C \succ D$ *iff for all ground substitutions* σ, $C\sigma \succ D\sigma$.

1. *Prove that is* $\succ$ *is a reduction order on the ground clauses, then it is a reduction order on the non-ground clauses.*

2. *Prove that if it is a total reduction order on the ground clauses, then it is a simplification order on the non-ground clauses.*

Exercise 22 *Prove or disprove:*

1. *The inference system based on* $\mathcal{B}$ *is a decision procedure for the ground clauses;*

2. *The inference system based on* Ordered $\mathcal{B}$ *is a decision procedure for the ground clauses.*

Exercise 23 *A Horn clause is a clause with at most one positive literal. Prove the refutation soundness and completeness of the following rule for Horn clauses:*

Positive Unit Resolution:

$$\frac{A \quad D \vee \neg B}{D\sigma}$$

where $\sigma = mgu(A, B)$.

Exercise 24 *Unit Resolution is a binary resolution where one of the premises must be a unit clause (i.e. a clause containing one literal). Using Unit Resolution prove that* $\exists x_5.\neg S(x_5) \wedge R(x_5)$ *follows from the following set of universally quantified clauses:*
$\{\neg S(x_1) \vee \neg P(x_1, b),\ \neg P(a, x_2) \vee Q(x_2, x_2),\ \neg Q(x_3, y) \vee R(x_3) \vee S(x_3),\ P(a, x_4)\}$.