Superposition with selection (1)

Superposition right:

$$\frac{\Gamma' \rightarrow l \approx r, \Delta' \quad \Gamma \rightarrow s \approx t, \Delta}{\Gamma'\sigma, \Gamma\sigma \rightarrow s[r]_p\sigma \approx t\sigma, \Delta\sigma, \Delta'\sigma}$$

where:

- $s|_p$ is not a variable, $\sigma = mgu(s|_p, l)$,
- $l\sigma$ is strictly maximal among terms in $\Gamma'\sigma \cup \{r\sigma\}$ and $l\sigma \approx r\sigma$ is strictly maximal with respect to the equations in $\Delta'\sigma$,
- nothing is selected in $\Gamma'\sigma \rightarrow l\sigma \approx r\sigma, \Delta'\sigma$
- $s\sigma$ is strictly maximal among terms in $\Gamma\sigma \cup \{t\sigma\}$ and $s\sigma \approx t\sigma$ is strictly maximal with respect to the equations in $\Delta\sigma$
- nothing is selected in $\Gamma\sigma \rightarrow s\sigma \approx t\sigma, \Delta\sigma$

Superposition with selection (2)

Superposition left:

$$\frac{\Gamma' \to l \approx r, \Delta' \quad \Gamma, s \approx t \to \Delta}{\Gamma'\sigma, \Gamma\sigma, s[r]_p\sigma \approx t\sigma \to \Delta\sigma, \Delta'\sigma}$$

where:

- $s|_p$ is not a variable, $\sigma = mgu(s|_p, l)$,
- $l\sigma$ is strictly maximal among terms in $\Gamma'\sigma \cup \{r\sigma\}$ and $l\sigma \approx r\sigma$ is strictly maximal with respect to the equations in $\Delta'\sigma$,
- nothing is selected in $\Gamma'\sigma \to l\sigma \approx r\sigma, \Delta'\sigma$
- $t\sigma \not\succ s\sigma$
- $s\sigma \approx t\sigma$ is selected in $\Gamma\sigma, s\sigma \approx t\sigma \to \Delta\sigma$ or nothing is selected in $\Gamma\sigma, s\sigma \approx t\sigma \to \Delta\sigma$ and $s\sigma \approx t\sigma$ is maximal with respect to the equations in $\Gamma\sigma \cup \Delta\sigma$

Superposition with selection (3)

Equality Resolution:

$$\frac{\Gamma, s \approx t \to \Delta}{\Gamma\sigma \to \Delta\sigma}$$

where:

- $\sigma = mgu(s, t)$,
- $s\sigma \approx t\sigma$ is selected in $\Gamma\sigma, s\sigma \approx t\sigma \to \Delta\sigma$ or nothing is selected in $\Gamma\sigma, s\sigma \approx t\sigma \to \Delta\sigma$ and $s\sigma \approx t\sigma$ is maximal in $\Gamma\sigma \cup \Delta\sigma$.

Equality Factoring:

$$\frac{\Gamma \to s \approx t, s' \approx t', \Delta}{\Gamma\sigma, t\sigma \approx t'\sigma \to s\sigma \approx t'\sigma, \Delta\sigma}$$

where:

- $\sigma = mgu(s, s')$,
- $s\sigma$ is strictly maximal among terms in $\Gamma\sigma \cup \{t\sigma\}$ and $s\sigma \approx t\sigma$ is maximal with respect to the equations in $\Delta\sigma \cup \{s'\sigma \approx t'\sigma\}$,
- nothing is selected in $\Gamma\sigma \to s\sigma \approx t\sigma, s'\sigma \approx t'\sigma, \Delta\sigma$.